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Evaluation of Probability Mass Function of Starting States of Flow Process of Hypogeometrical Early Arrival Discrete-Time DSR Load Management System

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Abstract: Flow process is very effective technique to trace the flow path and time of customer when it arrives in the system. In this paper, we develop and evaluate the dual service rate load management system through hypogeometrical early arrival discrete time system. The flow process Markov chain of the system is constructed through system state and service process Markov chains which depicts the all flow paths and time of customer which can be entered at any starting state till the customer leave the system. The arrival process of the system is modeled by a two phase discrete time hypogeometrical distribution and service process is depends on the system size. We compute the starting states probability mass functions and total probability mass function to show the flow time of customer. For numerous values of arrival and service probabilities, analytical results obtained and they are validated through simulation results.

Keywords: Early arrival system, Flow process, Probability mass function, Dual service rate

1. INTRODUCTION

Flow time calculation of customer in any system is most important characteristic metric. The flow time process is the process of observing and analyzing the total time spent by the customer in the system (Bocharov, 1996). The flow time calculation also includes the time when customer enters and leaves the system. It is very efficient and advantageous technique to analyze the total stay and path of customer in the system which follows from its entrance to the leaving. The process of flow time is determined by analyzing through the test customer and it is influenced by the already present customers in the system (Asif, 2010).

In most of the systems, load management is the typical requirement to avoid the congestion in the system which creates due to the heavy load of customers especially in peak times (Asif 2009). It can be done by varying the service speed of the system at the peak time. To avoid the congestion in the system by employing the load management mechanism through the service rate and it can be efficiently modeled through queueing system in discrete domain.

Due to the advancement of technology, most of the systems are discrete in nature and they require the analysis of system in discrete domain (Hans, 2001). Therefore, the continuous time queueing system is not efficient to evaluate the system performance. Discrete time queueing system is the efficient way to analyze any digital system due to its discrete nature (Peixia 2004, Gupta, 2000). It utilizes the concept of time slot where time is divided into equidistance slots. Discrete time queueing system can be classified into two systems according to the arrival of customer in the slot. These systems are early arrival system (EAS) and late arrival system (LAS).

Any digital system where arrival of customers needs to cross several stages to completely enter the system can efficiently be modeled by a discrete time phase type distribution. Stages in phase type distribution are inter-related with geometric distribution (Johnson, 1993). Discrete time hypogeometrical distribution is the most important phase type distribution and we use this distribution to model the arrival process of the system.

In this paper, we calculate and analyze the probability mass function of each starting state and total probability mass function of flow process of hypogeometrical early arrival discrete time load management queueing system using dual service rate.

The organization of this paper is as follows: Early arrival discrete time system, discrete time phase type distribution and flow process are briefly discussed in Section 2. In Section 3 hypogeometrical early arrival discrete time load management queueing system through dual service rate and its system state Markov chain, service process and flow process Markov chain and its starting probabilities are discussed. Finally the results and conclusions are presents in Sections 4 and 5 respectively.

2. MATERIAL AND METHOD

1) Early arrival system

Early arrival discrete time queueing system based on the slotted time operation where arrival occurs just beginning of the slot. The queue length increases when no departure or service takes place in a same slot.

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The timing diagram of the early arrival system is shown in (**Fig. 1**).

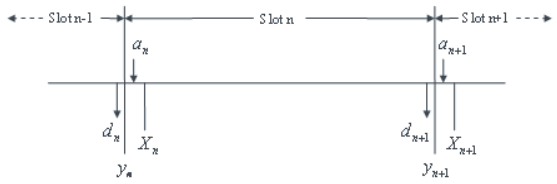


Fig. 1: Early arrival system

2) Discrete time phase type distribution

If one or more probability distribution of system is inter-related in a sequence then that distribution is called phase type distribution. When probability distribution is a geometrical then the distribution is a discrete time phase type distribution. The (**Fig. 2**). shows the discrete time hypogeometrical phase type distribution in which phases are in series.

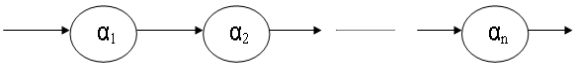


Fig. 2: **Discrete-time hypogeometrical distribution**

3) Flow Time Process

Flow time process is also called the life time process and it can be represented by constructing the flow time process Markov chain. It provides the graphical representation of over all system flow of each customer, when it enters and leaves the system. The flow process depicts the all possible flow paths and states where customers are allowed to enter the system and begins it flow and the states where customers leave the system. The flow process Markov chain is constructed through the system state Markov chain and service process Markov chain. The probabilities of starting states, where customer can be enter in the system and starts its flow, can be calculated by the probability mass function.

4) Hypogeometrical early arrival discrete time load management queueing system using dual service rate.

The thresholds based finite discrete time load management queueing system is shown in (**Fig. 3)** In this system, arrival process of customers is modeled by two phase discrete time hypogeometrical distribution with probabilities α_1 and α_2 and service process is modeled by geometric distribution. The system has two internal thresholds which manage the load in the system called S_1 and S_2. This system provides two types of service probabilities, β_n and β_f, which are totally depends on the internal thresholds of the system.

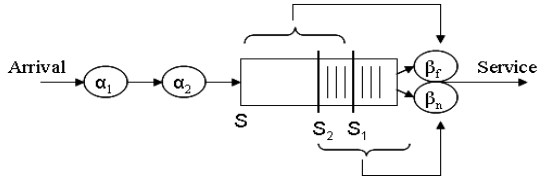


Fig. 3. **Queueing system**

A. ***System state Markov chain***

The system state Markov chain of the system is constructed as an early arrival system as shown in (**Fig. 4)**. The states of the Markov chain are represented by two variables separated by commas, (*i* , *j*), where I represents the number of customers in the system and j represents the phase of arriving customer. The states (0,1) and (0,2) represents the idle with phase 1 and 2 respectively. The system state changes from idle state to state (1,1), when arriving customer passes both phases. The system state (3,1) represent the condition of system where customers are served through normal probability of service β_n. When the system size reaches to the system state (4,1) the normal probability of service β_n switched to the faster probability of service β_f. Similarly when the system size is reduced and less then the system state (4,1) the faster probability of service β_f switched back to the normal probability of service β_n. The system state (5,2) represents the blocking state of the system where system is full and not allowed the any customer if the not departure takes place in the slot.

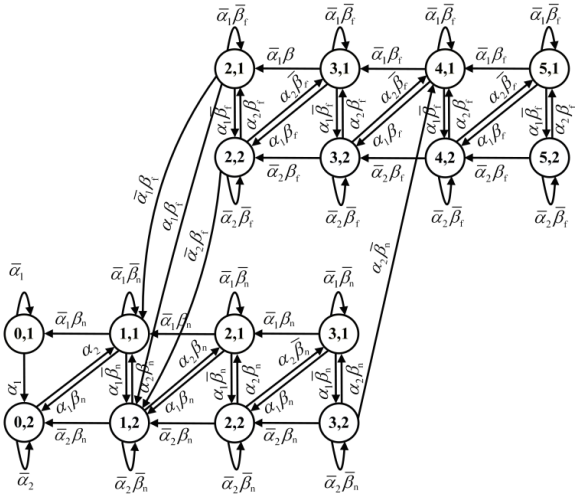


Fig. 4: System state Markov chain

B. ***Flow process Markov chain***

The Flow process Markov chain and service process Markov chain are shown in (**Fig. 5)**. The service process of Markov chain shows that the number of customers in a system is decreases when the customer is departing from the system after receiving the service or remains same if no customer is departed. The flow process Markov chain is constructed by utilizing the system state and service process Markov chain. The horizontal and vertical states of flow process Markov chain represent system states and service process of the system respectively. The customer flow process is represented by inclined states of flow process Markov chain. Three different states in flow process Markov chain can be distinguished according to the flow of customer:

1. Starting/direct states 2. Indirect states 3. Absorbing states

In starting states, customers are allowed to begins it flow process and these states are 17 (110), 31 (211), 32 (211) , 43 (312), 44 (312), 51 (413)and 55 (514). The states from 1 to 16 are absorbing states where customer leave the system and rest of the states are indirect states where customers are not allowed to enter the system directly and they could not begins their flow process. Flow process time of customer can be shorter or longer depends on the state where it enters the system.

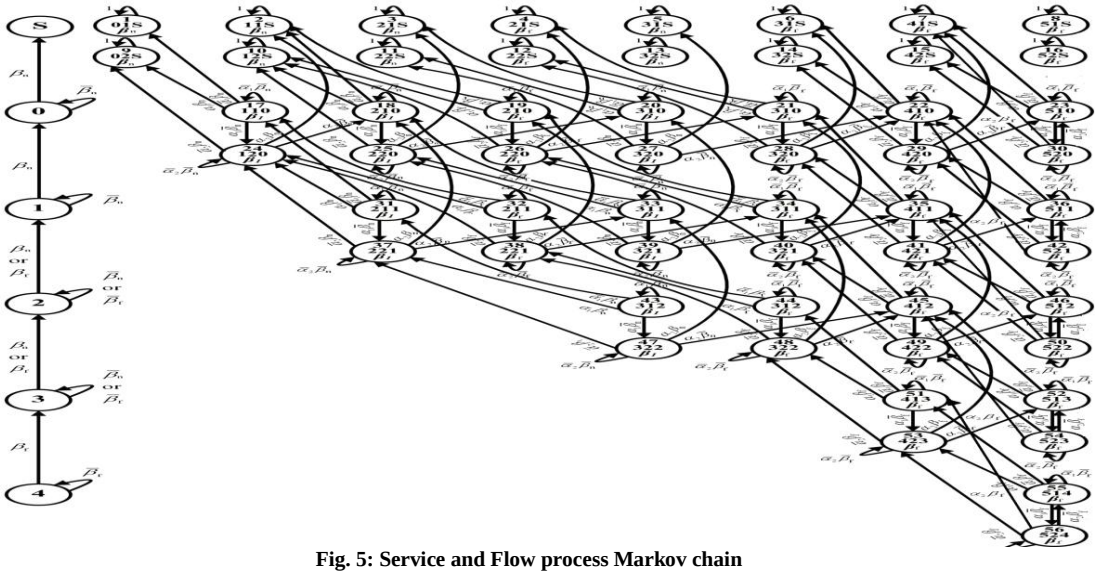


Fig. 5: Service and Flow process Markov chain

C. *Probability mass function of starting states of flow process*

There are seven starting states in a flow process where customers can begins its flow and these starting states are 17 (110), 31 (211), 32 (211) , 43 (312), 44 (312), 51 (413)and 55 (514). The probability mass function of each starting state of flow process can be calculated from Equations 1 to 7.

P[17(110)]

$$P_{(17)}^{\phi}=\left[\frac{p(2)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}+\frac{p(4)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+|\,p(14)+p(16)}\beta_n\right] \quad (1)$$

$$\times\frac{1}{1-\dfrac{p(16)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}}$$

P[31(211)]

$$P_{(31)}^{\phi}=\left[\frac{p(4)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}\ddot{\beta}_n+\frac{p(6)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}\beta_n\right. \quad (2)$$

$$\left.+\frac{p(8)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}\beta_f\right]\times\frac{1}{1-\dfrac{p(16)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}}$$

P[32(211)]

$$P_{(32)}^{\phi}=\left[\frac{p(8)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}\beta_f\right]\times\frac{1}{1-\dfrac{p(16)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}} \quad (3)$$

P[43(312)]

$$P_{(43)}^{\phi}=\left[\frac{p(6)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}\dddot{\beta}_n+\frac{p(10)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+|\,p(14)+p(16)}\beta_n\right] \quad (4)$$

$$\times\frac{1}{1-\dfrac{p(16)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}}$$

P[44(312)]

$$P_{(44)}^{\phi}=\left[\frac{p(8)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}\ddot{\beta}_f+\frac{p(12)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+|\,p(14)+p(16)}\beta_f\right] \quad (5)$$

$$\times\frac{1}{1-\dfrac{p(16)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}}$$

P[51(413)]

$$P_{(31)}^{\phi}=\left[\frac{p(10)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}\ddot{\beta}_n+\frac{p(12)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}\ddot{\beta}_f\right.$$
$$\left.+\frac{p(14)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}\beta_f\right]\times\frac{1}{1-\dfrac{p(16)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}} \quad (6)$$

P[55(514)]

$$P_{(55)}^{\phi}=\left[\frac{p(14)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}\ddot{\beta}_f+\frac{p(16)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+|\,p(14)+p(16)}\beta_f\right] \quad (7)$$
$$\times\frac{1}{1-\dfrac{p(16)}{p(2)+p(4)+p(6)+p(8)+p(10)+p(12)+p(14)+p(16)}}$$

Where p(.) shows the probability of the system state.

3. RESULTS AND DISCUSSION

An analytical and simulation program for calculating the probability mass function of each starting state and total probability mass are written in visual C++ and MATLAB. **(Fig. 6)** shows the probability mass function of each starting states and total probability mass function for values of arrival and service probability, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_n = 0.3$ and $\beta_f = 0.5$. It shows the flow process time of customer which entered at different starting states and it can easily be shown that the flow time is shorter for that customer which enter in the idle system which describe by the pmf of state 17. Similarly the customer entered at the state (31) finds one customer already present in the system, so the flow process time is comparatively larger than the customer entered at the state (17). The flow process time is longer for the customer which entered at the state (56). The total probability mass function is represented by the solid line, which shows the total flow time of customer. (**Fig. 7)**.

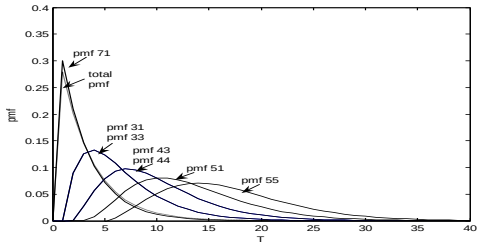


Fig. 6: **Probability mass function of starting states and total probability mass function for $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_n = 0.3$ and $\beta_f = 0.5$.**

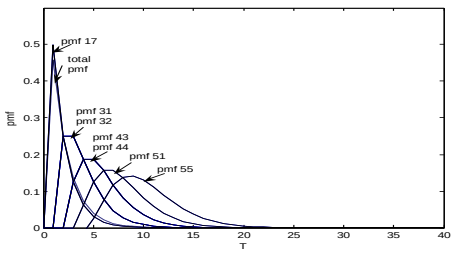


Fig. 7: Probability mass function of starting states and total probability mass function for $\alpha_1 = 0.3$, $\alpha_2 = 0.4$, $\beta_n = 0.5$ and $\beta_f = 0.6$.

4. CONCLUSION

In this paper, we used early arrival discrete time technique to model the dual service rate load management system. The arrival process of the system is modeled discrete time hypogeometrical distribution.

The flow time Markov chain was constructed from system state and service process Markov chains to show the flow process of customer in the system. The starting states probability mass function was calculated where customer can be entered in the system and begins its flow.

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