



A Computer Simulation of Flow of Newtonian Fluid through Backward Step Channel

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Abstract: The incompressible laminar flows of Newtonian fluid in sudden expansion channel are investigated. To seek the steady-state solution, a finite element time-marching algorithm, a so called, semi-implicit Taylor-Galerkin/pressure-correction scheme is employed. In contrast to vortex enhancement in expansion channel with increasing inertia suppression of recirculation region in both salient corner and the lip of the expansion is observed in expansion channel. The numerical predictions are compared with the experimental and other numerical results available in literature. The vortex length, recirculation flow rate and surplus pressure reduction in the expansion geometry are considered to analyse the consequences of fluid inertia.

Keywords: Expansion flows, Finite Element Method, Newtonian Fluids.

1. INTRODUCTION

The steady-state laminar flow in a sudden rectangular expansion channel using finite element method is investigated. Expansion flow is an interesting research problem due to presence of many important flow phenomena such as presence of singularities and appearances of lip vortices, growth of salient corner vortices, trailing edge vortices, and transitions all occur in the same closed geometry. Two and three dimensional sudden expansion flow is probably one of the standard benchmark problems used for the testing and the analysis of a numerical method due to complex geometry. Although it is impossible to obtain an analytical solution, it is easy to make some comparisons with available experimental data and numerical results. The flow is also relevant to a number of industrial and engineering applications like in refrigeration, extrusion, free jets and etc. (Baloch, *et al.*, 1994, Manica, *et al.*, 2003, Hassanipour, *et al.*, 2007 and Mitsoulis, *et al.*, 2009).

The extensive studies of laminar flow of Newtonian fluid in both two and three dimensional sudden expansion channels conducted and presented by many researchers only few are cited here (Hung, *et al.*, 1966, Boger, and Walters, 1993, Baloch, *et al.*, 1994 and Townsend and Webster, 1997). In two dimensional simulations the impact of inertia is investigated for the Reynolds number up to 65, whilst, in three dimensional geometry up to 200 (Baloch, *et al.*, 1994). The influence of distinct expansion ratios and the consequences of spatial dimension on vortex enlargement in form of vortex size, recirculation flow rate and loss of pressure differential have been investigated. For full incompressible Navier-Stokes equations adopting time marching semi-implicit Taylor-Galerkin/pressure-

correction scheme the numerical predictions are compared with the experimental results (Boger and Walters, 1993 and Townsend, and Walters, 1993). For expansion flows, the functional relationship has been derived between fluid inertia and longitudinal vortex cell size fitting the empirical data. In two dimension, through numerical results proved that with increase inertia and in small expansion ratios, the vortex cell size and intensity of eddy is increased in salient corner, whilst, with increase in backward step ratios, a duo of lip vortices occurs when the vortices move forward from re-entrant corner. At small expansion ratios, for both two and three dimension, the flow development is similar and at high expansion ratios in three dimensions, recirculation development starts at the lip of expansion as a duo of lip vortices near the backward step plane and with increase Reynolds number recirculation region expands to occupy the whole salient corner regions (Baloch, *et al.*, 1994).

The objective of the present work is to investigate the flow phenomena as observed in (Hassanipour, *et al.*, 2007) and study the influence of the inertia as increasing Reynolds number on expansion ratio, excess of pressure drop and to identify the basic vortex structure in the sudden expansion channels (Hassanipour, *et al.*, 2007 and Boger and Walters, 1993).

2. Governing System of Equations

Consider the laminar and incompressible flow of Newtonian fluid with vanishing of body forces, the flow governed by the following set of partial differential equations (PDEs) in a Cartesian coordinate system, conservation of mass and momentum transport equation is:

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

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$$\rho \frac{\partial \mathbf{u}}{\partial t} = \mu \nabla^2 \mathbf{u} - \rho (\mathbf{u} \cdot \nabla) \mathbf{u} - \nabla p \quad (2)$$

Where $\mathbf{u}$ and p are defined as velocity vector field and pressure over space and time, and $\frac{\partial \mathbf{u}}{\partial t}$ represented as

temporal derivative. Material parameters ρ and μ are given via density and viscosity respectively. To rewrite the governing equation (1) in dimensionless form so choosing non-dimensional variables corresponding to the space, fluid velocity, pressure and time as x^* , v^* , p^* and t^* are well-defined by appropriate scales as follows; $x = Lx^*$, $u = Uu^*$, $p = \rho U^2 p^*$ and

$$t = \frac{L}{U} t^*. \text{After substitutions the above dimensionless}$$

variables yields the non-dimensional generalised system of equations that may be stated as:

Continuity equation

$$\nabla \cdot \mathbf{u} = 0 \quad (3)$$

Momentum transport equation

$$\frac{\partial u}{\partial t} = \frac{1}{\text{Re}} \nabla^2 u - u \nabla \cdot u - \nabla p \quad (4)$$

$u(x, t),$ $P(x, t)$	Velocity vector, Isotropic pressure (per unit density)
D, t	Rate of strain tensor, time
∇, μ_0 and Re	Spatial differential operator, Constant fluid viscosity of Newtonian fluid and Reynolds number (Fluid inertia)

Stage – 1a: Given v^n and p^n , find $v^{n+1/2}$ such that:

$$\left\{ \left(\frac{2}{\Delta t} \left(v^{n+1/2} - v^n \right), v \right) + \frac{\mu(\varepsilon)}{2 \text{Re}} \left(\nabla \left(v^{n+1/2} - v^n \right), \nabla v \right) \right\} \left(p^n - \frac{1}{\text{Re}} \nabla v^n, \nabla v \right) - \left(((v, \nabla) v)^n, v \right) \quad (10)$$

Stage – 1b: Given v^n , $v^{n+1/2}$ and p^n , find v^* such that:

$$\left\{ \left(\frac{1}{\Delta t} (v^* - v^n), v \right) + \frac{1}{2 \text{Re}} \left(\nabla (v^* - v^n), \nabla v \right) \right\} \left(p^n - \frac{1}{\text{Re}} \nabla v^n, \nabla v \right) - \left(((v, \nabla) v)^{n+1/2}, v \right) \quad (11)$$

Stage – 2: Given v^* and p^n , find $p^{n+1} - p^n$ such that:

$$\left(\theta \nabla (p^{n+1} - p^n), \nabla q \right) = \frac{-1}{\Delta t} \left(\nabla v^*, q \right) \quad (12)$$

Stage – 3: Given v^* and $p^{n+1} - p^n$, find such that:

$$\left(\frac{1}{\Delta t} (v^{n+1} - v^*), v \right) = \left(\theta (p^{n+1} - p^n), \nabla v \right) \quad (13)$$

$$\text{Re} = \frac{\rho U L}{\mu_0} \quad (5)$$

whilst U is a characteristics velocity and L is a length scales (half channel height) to be considered. The concept of numerical algorithm, fully discrete system and definition of matrices is given in detail and can be found in (Baloch, *et al.*, 1994 and Townsend, and Webster, 1987).

3. Numerical Scheme

A time-marching semi-implicit Taylor–Galerkin/pressure-correction numerical scheme is adopted. The scheme is based on a fractional-staged approach, which discretises first in the temporal domain through Taylor series expansions and incorporates a two-step Lax–Wendroff approach to capture second order accuracy in time. This scheme displays a significant enlargement in accuracy and stability corresponding to pair of numerical schemes as finite difference and finite element schemes proposed by (Donea, 1984) for the transient solution of viscous incompressible flows. After that, the specific modification of an operator splitting technique a so-called pressure-correction method is adopted. This technique was developed by (Van Kan, 1986) that advances 2nd order overall correctness and strength by a linearized energy analysis.

Historically, these schemes proposed by (Hawken, *et al.*, 1990) in the explicit form and then recognised by (Townsend and Webster, 1987), who established scheme as Taylor–Galerkin/Pressure-Correction (TGPC) for the flow of viscoelastic fluids to achieve the second-order accuracy. The Taylor–Galerkin algorithm has three distinct fractional stages per time step.

Upon restriction to finite dimensional subspaces V^h , V_0^h and p^h of v , v_0 and p respectively, to arrive at the fully discrete statement of the problem. For the velocity, piecewise quadratic functions and for pressure, piecewise linear functions are adopted and triangular meshes are employed in two dimensions on the spatial domain ' Ω ' (Baloch, *et al.*, 1994).

4. Problem Definition and Mathematical Formulation

Consider two-dimensional laminar flows of an incompressible Newtonian fluid from a sudden expansion channel of specific 1:4 ratio, relevance to engineering applications such as processing industry. Two long channels of very large same width are connected with each other providing two dimensional expansion planes. The channel length for upstream is $49L$ and downstream is taken as $27.5L$, where L is an inflow characteristics length as half upstream channel height as shown in (Fig. 1). Whereas, the height of the small half channel is L and large channel is $4L$. In two dimensional geometry total numbers of elements, total number of nodes and degrees of freedom are 2987, 6220, 14057 respectively. The numerical results are compared with other numerical, as well as, experimental results (Baloch, *et al.*, 1994, Hassanipour and Jose. 2007 and Boger and Walters, 1993). The triangular elements are chosen for the domain.

Fully developed at both inlet and outlet parabolic velocity boundary conditions are imposed to satisfy the Dirichlet boundary conditions and no-slip boundary conditions are fixed on the boundary walls. Whilst, at the outlet, traction free boundary condition with zero pressure datum is imposed. However, at axis of symmetry mixed i.e., Neumann and Dirichlet boundary conditions are fixed with vanishing cross component of velocity. These conditions prove to be good for convergence. The predictions start with quiescent initial conditions at lower value of the Reynolds number. For higher values of inertia steady-state solution are realised in a continuum approach.

Taylor–Galerkin /Pressure–correction scheme in different context has been discussed in the literature (Baloch, *et al.*, 1994, Townsend and Walters, 1987 and 1993). Equations (3) and (4) are supplemented by appropriate initial and boundary conditions, the initial condition for the full Navier–Stokes problem consists of specifying the value of $\mathbf{u}$ at the initial time:

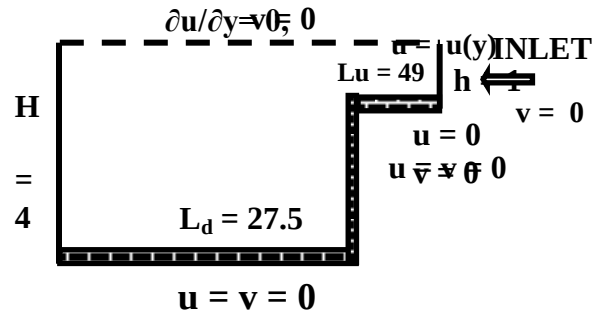


Fig.1: Schematic diagram 1:4 planner expansion channel with boundary conditions.

L_d = Length of Down Stream, L_u = Length of Up Stream and H = Height

Conditions	Velocity	Pressure
Initial Conditions	$\mathbf{u}(\mathbf{x}, t) = \mathbf{u}_0(\mathbf{x}, 0)$	$p(\mathbf{x}, t) = p_0(\mathbf{x}, 0)$
Boundary Conditions	1. At solid wall: no-slip $u = v = 0$ 2. At exit: axial velocity u is free and vanishing cross flow velocity ($v = 0$). 3. At axis of symmetry: $v = 0$ and Traction free condition $\frac{\partial u}{\partial y} = 0$ 4. At Inlet: $\mathbf{u} = V_{\max} (y-a)(b-y)$	At exit $p = 0$

The interesting flow feature of the investigation is the total pressure drop across the geometry. An empirical relationship has been developed, that allows estimating the pressure drop from analytically obtained combined pressure drop in both channels of a 1:4 sudden expansion. The following relationships for Couette–correction (C) have been used to obtain pressure drop that relates with the flow characteristics in the domain.

$$C = \frac{\text{Re } \delta p - (L_u \nabla P_u + L_d \nabla P_d)}{2\tau_w} \quad 14$$

Where Re and δP is a Reynolds's number and is a numerically predicted total pressure loss in the geometry, L_d and L_u are downstream and upstream lengths, whilst, ∇P_d and ∇P_u are downstream and upstream pressure gradient respectively computed

analytically, and τ_w is the wall shear stress $\tau_w = \left. \frac{\partial \mathbf{u}}{\partial y} \right|_w$ in

the fully developed downstream flow. For simulations a finite mesh is employed i.e., refined (M) to validate the algorithm and displayed in (Fig. 2),

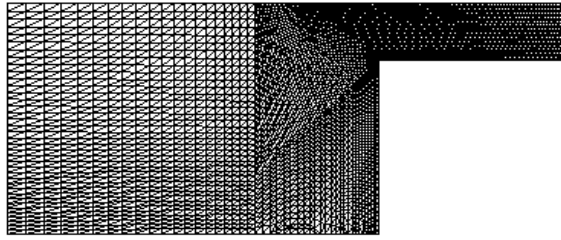


Fig. 2: Refined Finite Element Mesh (M) for two dimensional 1:4 geometry

5. NUMERICAL RESULTS AND DISCUSSION

The numerical predictions are presented through streamlines contours to express the flow structure and pressure drop vindicated through isobars. The flow structure analysed via salient corner vortex length recirculation flow rate and excess pressure drop in the domain to simplify comparison of the effects of Reynolds number (fluid inertia) and presence of porous material. The numerical results are compared with available experimental results (Townsend, and Walters, 1987 and 1993) different numerical results that are presented in a literature.

5.1 The impact of fluid inertia:

(a) Streamline Contours for two dimensional 1:4 Expansion Flow of Newtonian Fluid

The computed steady-state solutions are presented through streamlines contours and pressure isobars. The numerical results-oriented are in two dimensional 1:4 expansion with increasing fluid inertia (Re) from 0.001 to fifty. The predictions are clearly described that at low value of Reynolds number a small salient corner vortex activity starts creating recirculating regions and subsequently, as Re values increases vortex enhancement follows. At $Re = 0.001$, the fluid goes in the expansion and spread out, leaving fairly important stagnant regions in each salient corner. Once the fluid comes near the expansion plane it is incapable to exchange the sharp corner as it leaves the wall. The same size of stagnant (motionless) regions ensues with increasing Reynolds number up to fifty, nevertheless, a salient corner vortex grows in size, and intensity, subsequently, vortex enhanced and fills the entire region.

For the two-Dimensional 1:4 expansion channel the predicted streamline contours are displayed in (Fig. 3), with increasing Reynolds number and presented in two regions i.e., main core flow region and recirculation region at fixed incremental values. For each Reynolds number, thirteen (13) contours are selected in main core flow, whilst in recirculating region according to

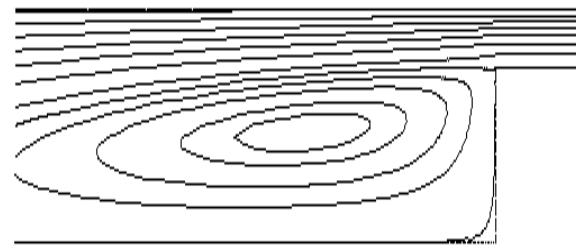
intensity different contour values are selected for each Reynolds number with equal increments of non-dimensional stream function value from minimum to maximum, such as for $Re = 0.001$ the contour values are 0.00001, 0.0001 and 0.0007.



(a) $(Re, Q_v) \equiv (0.001, 0.00077006)$;



(b) $(Re, Q_v) \equiv (1, 0.00157)$



(c) $(Re, Q_v) \equiv (50, 0.20929)$

Fig.3: (a – c): Stream lines for 1 : 4 expansion flow

Another interesting flow feature is the behaviour of the recirculating flow rate Q_v , described as a ratio between the differences of stream function from the main core flow to the vortex centre. The numerical results of the recirculation flow rate with increasing inertia are displayed through graph plotted in (Fig. 4), that demonstrate the size of vortex intensity is linearly increasing with increasing Reynolds number. Consequently, the vortex cell size i.e., separation and reattachment vortex length with increasing inertial values is presented in a (Fig. 5). From figure it is illustrated that the vortex lengths increase continuously in linear fashion with increasing Reynolds number. All predicted numerical results are closely in agreement to (Hung, and Macagno, 1966, Macagno, and Hung, 1967 Durst, et al. 1974 and Baloch, 1994). The empirical relationship of the vortex cell size (X) with increasing Reynolds number is described by fitting the computed data, as:

$$X = 0.3953Re + 1.1604 \quad 0.001 \leq Re \leq 50$$

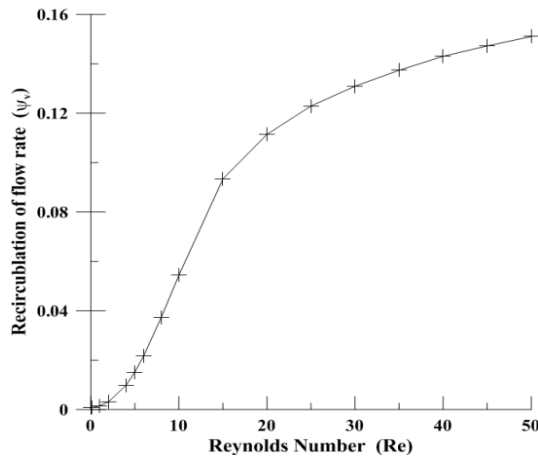


Fig. 4: Two dimensional expansion flows, recirculation flow rate (Vortex intensity)(Q_v) is a function of fluid inertia (Re)

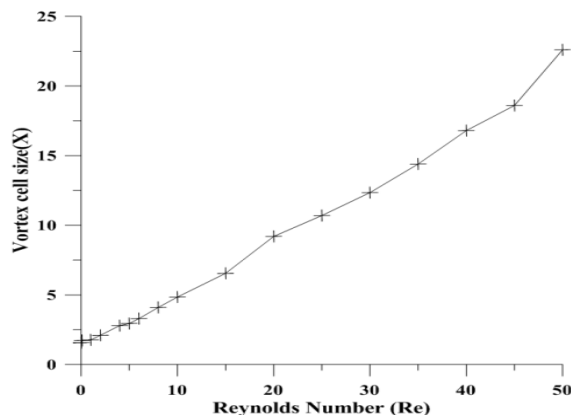


Fig. 5: Graph of the vortex cell size (X) is a function of Reynolds Number (Re)

6. Discussion on Pressure differentials and Couette – Correction

Pressure contours are described here at fixed incremental value taking 15 contours along the flow direction in the domain as for same as stream function. In figure (a to b), pressure isobars at different inertia ($Re=1$ to 50) are drawn so as to influence of pressure drop on different Reynolds number is to be studied. It is clearly shown that with increase Reynolds number the pressure isobars are increased and the quantitative value of pressure drop is linearly decreased. The minimum pressure values P_{min} are shown in figures (a to b) with increasing Reynolds number and asymmetric isobars are detected from $Re = 1$ to 50. The maximum and minimum value of pressure is detected of non-dimensional 100350.0 units, which characterises the pressure drop through the flow and -3496.6 non-dimensional units in the vicinity of expansion corner.

In (Fig-8), Couette–Correction (CC) is plotted whereas the data is presented in common logarithm forms to show the predicted results against figure 12 of Thin Swe for direct comparison. For the computation of

the Couette–Correction similar procedure is adopted and plotted as a function of Reynolds Number (Re). The qualitative trend of the figure 9 is same as Swe and quantitative values are different due to change in expansion ratio and co-ordinate system.

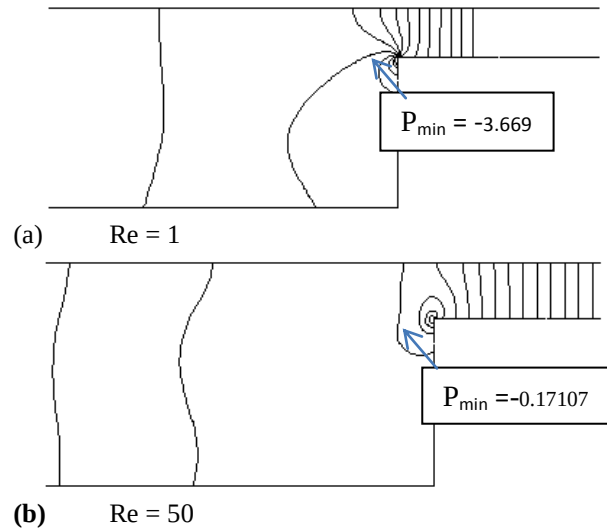


Fig. 6 (a to c): Graph of the Pressure contours with increasing fluid inertia

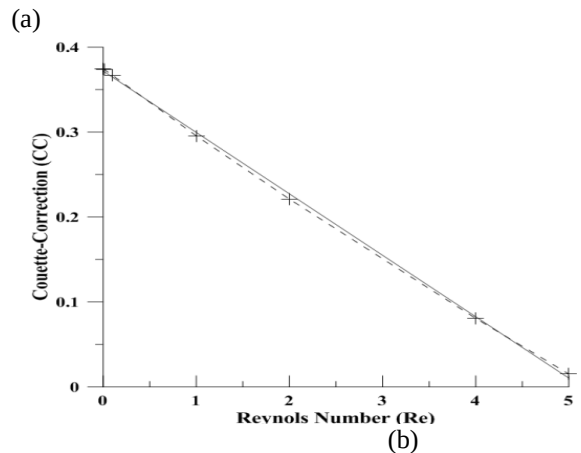


Fig. 7(a-b): Graph of the Couette-correction (CC) with increasing fluid inertia

The Couette–Correction at lower and higher inertial values are presented in **(Fig.–7(a))** for $Re = 0.001$ to 5 shows the “CC” is monotonically decreasing and in **(Fig.–7(b))** for $Re = 6$ to 50, displays monotonically increasing.

For Couette–Correction, (CC), the empirical relationship is developed by fitting the computed data in two limiting intervals, i.e., from $Re = 0.001$ to 5 and $Re = 6$ to 50 separately and the equations are presented below. The computed and fitted data is plotted in **(Fig–8)**.

$$C = 0.37199953 - 0.07231085Re \quad 0.001 \leq Re \leq 5$$

$$C = 0.0391Re - 0.1301 \quad 6 \leq Re \leq 50$$

Whilst, for direct comparison with [Thin Thin Swe] **(Fig–8)** is plotted.

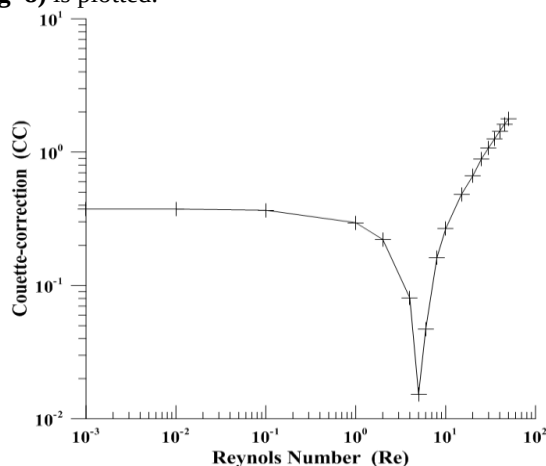


Fig.8: Couette–Correction (cc) is a function of fluid inertia (Re)

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CONCLUSION

The flow of an incompressible Newtonian fluid has been investigated through 1:4 sudden expansion channel. The numerical solutions are predicted through a so called, semi-implicit Taylor–Galerkin/pressure–correction scheme. For flow through channels, based on the numerical predictions, demonstrating the effect of fluid inertia on flow structure a good agreement is observed with other experimental and numerical solutions on vortex developments regarding vortex length and intensity, and excess pressure drop.

For both simulations, the expansion channel, the monotonically decreasing pressure differentials are observed.

7.

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