



Computational Modelling of Pressure Difference for Non-Newtonian Behaviour of Blood in Plaque Deposited Capillaries

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Abstract The impact of shear dependent functional viscosity of blood is studied by employing Power law and Carreau model, in a plaque deposited capillary segment. Numerical computations are carried out in terms of pressure differential. Predicated results are illustrated along non-dimensional pressure isobars at different inertia levels along various depositions taken in percentages in downstream of a capillary segment. Computed results are made via semi-implicit Taylor-Galerkin/pressure-correction scheme, which is termed as second order accurate time stepping scheme. Predicted and analytical results are very close on validation side. The predicated trend of pressure drop is observed in linear form increasing with deposition or inertia levels. The contribution of this work in future will be treated as a tool for research in cardio vascular diseases, planning for treatment and design of medical devices.

Keywords: Shear Thinning, Down Stream, Non-Newtonian, Vessels.

1. INTRODUCTION

Flow of blood in capillaries, relevance to atherosclerosis, causes severe cardiovascular diseases, as increasing level of blockage disturbs the flow field of blood, hence large number of deaths takes place every year. Hence hemodynamic factors play an important role in the cases, where deposition of plaque increases. Therefore, at present, researchers are engaged to study the hemodynamic factors related to the blood flow conditions in the capillaries.

From a rheological study, blood presents inconsistent behaviour, while flowing in large versus small vessels. In arteries, the behaviour of blood is considered as Newtonian, with the shear rate greater than 100 s^{-1} (Bonert *et al.*, 2002; Fei *et al.*, 1994), and most of the experimental and numerical work is based on that assumption. Whereas, in capillaries the behaviour of blood is assumed non-Newtonian having shear rate less than 100 s^{-1} , and the stresses depends nonlinearly on the deformation rate (Song *et al.*, 2000).

Most of the hemodynamic studies have been performed, to investigate the impact of inertia and deposition on the flow field of blood in capillaries, under the assumption of steady state. The distinction between water and blood, flowing along a capillary segment has been reported, with the states of Newtonian and non-Newtonian nature of blood (Mao and Hanratty, 1986). Whereas, in some investigations, viscosity of blood is presented for simulation by the mixture of water and glycerol (Takuji and Guimaraes, 1998). Most of the experimental as well as numerical studies, predicted by the assumption of axi-symmetric flow of blood (Forrest and Young, 1970).

2. GOVERNING SYSTEM OF EQUATIONS

The continuity and momentum transport equations have been employed for modelling laminar type flow of blood in a partially blocked capillary segment. The governing equations for cylindrical polar coordinates are written as:

Continuity Equation

$$\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{\partial v_z}{\partial z} = 0 \quad (1)$$

$$\rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} \right) = \left(\frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rz}) + \frac{\partial}{\partial z} \tau_{zz} \right) - \frac{\partial p}{\partial z} \quad (2)$$

Momentum Balance Equation in terms r and z components:

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$$\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} \right) = \left(\frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rr}) - \frac{\tau_{\theta\theta}}{r} + \frac{\partial}{\partial z} \tau_{rz} \right) - \frac{\partial p}{\partial r} \quad (3)$$

Where ρ denotes the density, p is the isotropic pressure, whilst, v_r and v_z are respectively velocity components in the radial (r) and axial (z) directions of the blood and t is time. Then $\tau_{rr}, \tau_{rz}, \tau_{zz}$ and $\tau_{\theta\theta}$ denotes the components of the extra stress tensor and defined as follows:

$$\tau_{rr} = 2\mu(\dot{\gamma}) \frac{\partial v_r}{\partial r}$$

$$\tau_{rz} = \mu(\dot{\gamma}) \left\{ \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right\}$$

$$\tau_{zz} = 2\mu(\dot{\gamma}) \frac{\partial v_z}{\partial z} \quad \tau_{\theta\theta} = 2\mu(\dot{\gamma}) \frac{v_r}{r}$$

Problem Specification And Boundary Conditions

To specify the problem, initial and boundary conditions are to be addressed as:

$$v(z, 0) = v_0(z), \text{ subject to } \nabla \cdot v_0 = 0.$$

The plaque deposited capillary segment, on assumption side is treated as an inflexible circular tube, whereas, capillary walls are assumed solid; therefore they are treated free from slip boundary conditions. The deposition is treated as in parabolic form, increasing in length as well in depth, along various Reynolds numbers. Axi-symmetric flow of blood is assumed; hence only upper half of the flow region is simulated. Neumann boundary conditions are employed on the axis of symmetry, whilst at out let zero pressure and traction free velocity components are imposed.

The analytical solution for Power law model (Dvinsky and Ojha, 1994):

$$V_z = \frac{1}{4} \frac{6n+2}{(n+1)} \left(\frac{1}{R_0} \right)^2 \left(1 - \left(\frac{r}{R_0} \right)^{\frac{n+1}{n}} \right), \quad 0 \leq r \leq R_0 \quad (4)$$

Where, R_0 is a non-dimensional radius of the artery in non-blockage region. Pressure difference at inlet along with boundary conditions is computed by employing equation (4) as:

$$p_{in} = \left[2k \left(\frac{3n+1}{2n} \right)^n z \right] \quad (5)$$

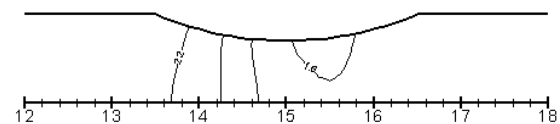
3. RESULTS AND DISCUSSIONS

Numerical results for pressure drop against different Reynolds numbers in a partially blocked capillary segment are computed by employing Power Law and Carreau model respectively. In **(Table-1)** a non-dimensional scaled pressure drop verses inertia levels being set at 100, 200 and 300 respectively, in terms Reynolds numbers, having deposition levels, taken in percentages as, 30, 50 and 70 respectively is presented. It is observed that scaled pressure drop for Power law model is smaller than Carreau model, whereas, Carreau model shows similar trend as Newtonian simulation. Further, it is observed that scaled pressure drop linearly increases as inertia or deposition level increases. It is clearly illustrated that constant scaled pressure drop having a unit value is observed, which confirms the analytical solution. Furthermore, at 30% deposition level the change in slope is very small; at 50% deposition level is moderate and at 70% deposition level overshoots. These trends are further confirmed by presenting pressure isobars in **(Figs-1-3 (a-c))**, for Power Law model for various inertia levels at different deposition levels. Empirical equations have been developed and listed in **(Table-2)**.

To validate the model, analytical and numerical results for pressure drop for Power law model computed in a capillary segment, free from deposition at various inertia levels is displayed along with numerical results in **(Table-3)**. It is observed that both the results (analytical and numerical) are identical, which confirms the validation of the model.

4. CONCLUSION

Analytical and numerical results in terms of pressure drop for Power law model in a straight capillary segment are observed comparable and identical, confirming the validation of the model. Furthermore, numerical results for Power law model are less in comparison of Carreau model and their trend is linear, along increasing level of deposition or Reynolds number.



(a)

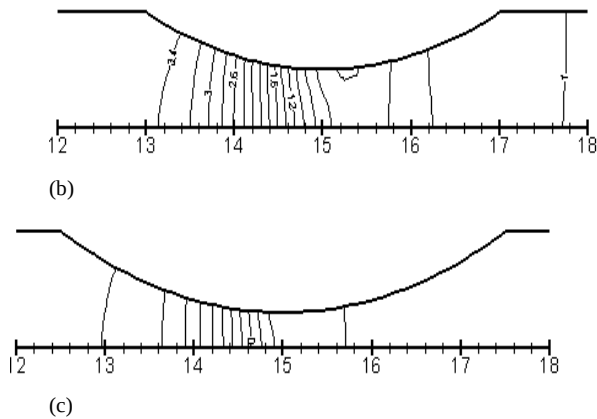


Fig. 1(a-c): Pressure isobars for Power law model at Reynolds number $Re = 100$ for different levels of deposition:

(a) 30% deposition; (b) 50% deposition;
(c) 70% deposition

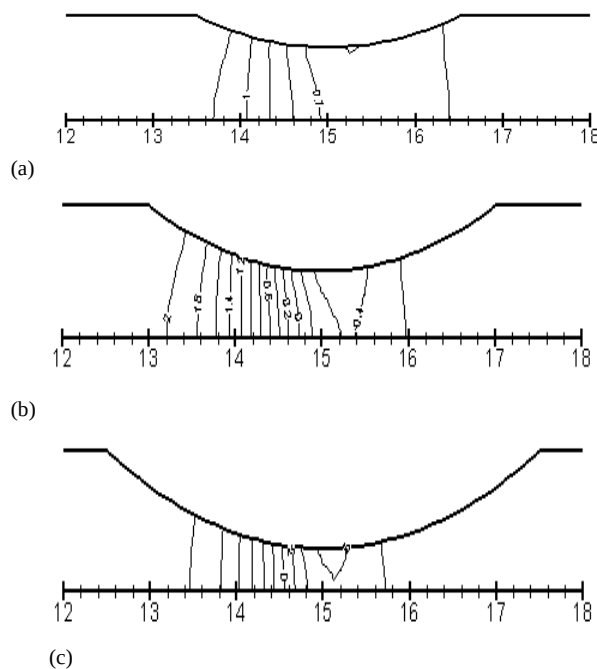


Fig. 2(a-c): Pressure isobars for Power law model at Reynolds number $Re = 200$ for different levels of deposition:

(a) 30% deposition; (b) 50% deposition; (c) 70% deposition

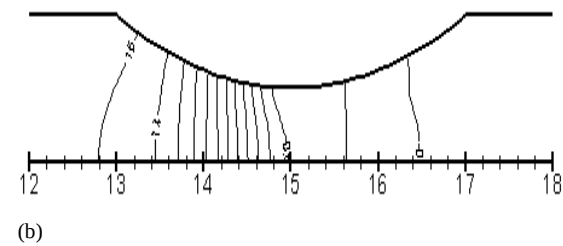
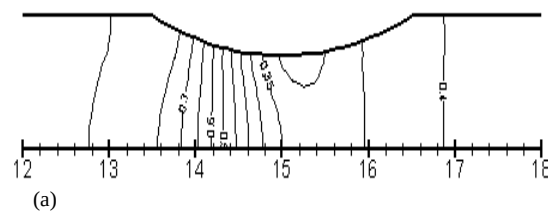


Fig. 3 (a-b): Pressure isobars for Power law model at Reynolds number $Re = 300$ for different levels of deposition:

(a) 30% deposition; (b) 50% deposition

Table-1 Scaled pressure drop against various levels of inertia for different percentages of deposition

Re	Scaled Pressure drop					
	30% deposition		50% deposition		70% deposition	
	Power	Carreau	Power	Carreau	Power	Carreau
100	1.108	1.283	1.5437	1.452	6.3947	7.931
200	1.111	1.309	1.808	2.195	10.623	13.23
300	1.129	1.324	2.059	2.510		

Table-2: Scaled pressure drop (P) for Power law model against Reynolds numbers for different percentages of deposition in terms of equations

Deposition	Equation
30%	$P = 0.0001 Re + 1.095$
50%	$P = 0.0025 Re + 1.306$
70%	$P = 0.0423 Re + 2.1664$

Table-3: Analytical and Numerical results of pressure drop for Power law model against various Reynolds numbers in a straight artery

Re	Analytical results	Numerical results	Error = $\frac{ A - N }{ A } 100$
01	248.27523	248.47004	0.08 %
50	4.9655	4.96504	0.00 %
100	2.48275	2.48098	0.07 %
150	1.65516	1.653165	0.12 %
200	1.24137	1.239374	0.16 %
250	0.993100	0.99388	0.08 %
300	0.82758	0.828233	0.08 %

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