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Estimation Of The Production Of Matiari Sugar Mill Sindh Using Spss Through Cobb-Douglas Model

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Abstract: The production function is the combination of the Labor and Capital. It is really a business concept that defines the maximum rate of output approaching from specified input rates of capital and labor. The least cost capital-labor combination for the production or the output rate would yield maximum profit, and are not the objectives of this function. This function only shows that the maximum output should be obtained from any input combination. Economists use variety of functional form to describe the function, but the most frequently used function is Cobb-Douglas production function. It was proposed by Charles Cobb and Paul Douglas in 1928. It is widely used in economics because it has good properties and is representative of much production process. Almost every production manager is interested in maximizing his production. He attempts to minimize the cost or maximize production, subject to producing a specified output rate. In this sense estimation of production plays the crucial role for planning maximum production but the main problem is which procedure of estimation should be used for the forecasting of production. This problem seriously depends on the behavior of production curve while Cobb-Douglas is a non-linear model and, therefore, the behavior of this model is curvilinear.

In this paper, our objective is to estimate the sugar production for the next two years (i.e., 2010, 2011). Due to the fixed capital[1] the production is mainly dependent on labor. That is why this paper also provides the optimal labor input as appendix information. All these analyses are carried by using the Statistical Package for Social Science (SPSS). This is widely used statistical package for the linear and non-linear regression analysis. The behavior of production is normally non-linear and can be analyzed through Cobb-Douglas function. This function consists of two independent variables i.e., labor and capital. In this software linear and non-linear regression analysis can be performed via Curve Estimation procedure, but Cobb-Douglas model is analyzed trough *Non-linear regression menu*. During the model building of Cobb-Douglas, starting values of parameters are essential to be given. In production function Return to Scale refers to a technical property of production that examines the changes in output following proportional changes in all inputs. Usually many functions have a property called constant return to scale which means an increase of an equal percentage in all factors of production causes an increase in output of the same percentage. In the SPSS during the modeling of production function the assumption of the constant return to scale is followed which is that the sum of parameters are exponents (i.e., α and β) on capital and labor variables must be equal to one. Estimation procedure makes iterations[1], and due to these iterations the model select the best parameters values for estimation.

In this study, estimated model for sugar production of Matiari Sugar Mill of Sindh for the collected data was found to be satisfactory with some diagnostic checks such as *standard deviation of regression* and *coefficient of determination (R-square)*; all these practices followed by SPSS.

Keywords: Production of Matiari Sugar Mill, Spss, Cobb-Douglas Model

1. INTRODUCTION

In the social science the mathematical and statistical analysis of actual economic phenomena based on the concurrent development of theory and observation is known as econometrics. Economical phenomena can be classified as micro and macro level, but the one of the most inevitable economical phenomena is production theory, which belongs to the both micro and macro level of economics. All businessmen as well as industrialists are very much concerned about the theory of the firm in order to make correct decisions about what item should be produced and how much and how to produce? All these situations can be handled by the awareness of economical production theory.

The production of the firm is mainly dependent upon the factors of production, which can be classified as capital and labor. The production is considered as explained or dependent variable, which depends on two independent variables: labor (L) and capital (K). Letting Y denote the amount of output of production, the production function can be written as;

$$Y = f(K, L). \qquad (1.1)$$

Generally the firm has to face the problem to determine how much labor and capital to employ to produce most efficient output, because every production manager wants to increase his production to the maximum point at the available factors of production. The appropriate utilization of labor and capital to maximize the production of the firm is the main objective of this paper. Ideally the production function is the microeconomics concept, which specifies the relationship between physical output and input. It is interesting to note that the production analysis of Matiari Sugar Mill is also a case of Microeconomics.

The equation (1.1) describes the general behavior of the production, which depends on labor and capital. The true relationship between the factors of production is likely to be close to Cobb-Douglas model. This model is widely used in theoretical and empirical analysis of production of the firm in the sense of microeconomics. The statistical concept of the estimation can be applied through Cobb-Douglas production function. This function predicts the production (output) using the different combinations of the factors of production (input). The computation of this model is not laborious and mostly production processes can be regressed by this model. This function does not imply directly to estimation by the simple regression method. It is a non-linear relationship and technically, the method of Ordinary Least Square (OLS)[1] is used on linear relationship, therefore the Cobb-Douglas function must be in a linear form by logarithmic transformation in order to use OLS.

Besides the advantages of Cobb-Douglas production function, some drawbacks are also faced by this function. One of the most important drawbacks is that, it cannot handle a large number of inputs. Serial correlation[2] and heteroscedasticity[3] are the common problem, which affect this function. The selected production function has constant shares of the factors of production over time which creates a doubt in analysis. Neither Cobb nor Douglas provided any theoretical reason why the coefficients α and β should be constant over time. Remember that the nature of machinery and other capital goods differ between the time periods. Similarly the skills of the labor also changes due to the changing of time. In the present study, not large numbers of inputs are used. Therefore, variations in the nature of capital and labor have not significant impact on production level and constant values of α and β are allowed to use in analysis.

Computer software makes our work easy to perform. In this paper the production function of Cobb-Douglas is performed by SPSS. It is most widely used program for statistical analysis in social science. According to this paper, the selected model is non-linear regression function, because the relationship between the dependent and independent variables is non-linear. This type of regression model in SPSS is performed in the *analysis menu* through *Non-linear Regression* option. The level of data in this option must be in quantitative (i.e., ratio scale) form. Therefore, the selected data for this paper belonging to the performance of the labor and capital is recorded in quantitative form.

The rest of the paper is organized as follows. In the next Section, we provided the comprehensive literature review about Cobb-Douglas production function. Section 3 provides the data information. In Section 4 the discussion about the methodology for the production estimation is given which includes the description about Cobb-Douglas model, SPSS, optimal employment of labor and some major diagnostic checks on regression are also defined in this section. Results of the estimation and discussion are presented in Section 4. Finally, conclusion is offered in Section 5.

2. Literature Review

Business concept of production from specified input rate of the capital and labor is known as Cobb-Douglas Production Model. Many studies have been carried out on this production function. Our literature review discusses the work on production function which has been done by different research scholars. Felipe and Adams (2005) observed that the Cobb-Douglas production function is found everywhere in the form of theoretical and empirical analysis of growth of productivity. They performed the comprehensive work on production model and conclude that the model has very serious implications for today's work in macroeconomics. Bhanumurthy (2002) argued that the Cobb-Douglas production has merits and it is viewed as a simple tool which can be handled easily. He discussed many merits and demerits of this function and found two main demerits, which were "it cannot handle large number of input" and "serial correlation and hetroscedasticity affects continually this function too" while the merits were that the production function facilitates computation and properties of parsimony and flexibility. Husain, *et al.,* (2010) studied Cobb-Douglas function. The objective of their study was to select the most suitable production model for measuring the production process of major different industries of Oman. Their estimated results suggested that major industries of Oman indicate Increasing Return to Scale. Felipe and McCombine (2003) attempted to evaluate the performance of aggregate production function. This production has been criticized by many scholars. In their study, they concluded that the hypothetical basis of the aggregated function is problematic. Edward, *et al.,* (2002) reviewed the relationship between labor and capital with the U. S. industry level. They accepted and put forward a concept that the true relationship between capital labor is likely to be close to Cobb-Douglas Model.

The study of Prajneshu (2008), belongs to the fitting of Cobb-Douglas Model. According to his, the

[1] Method of least square is used to estimate coefficients of model and provide best fitted line than other methods.

[2] Correlation between the members of series of observation with respect to time is called serial correlation and it is also called the autocorrelation.

[3] The variance of the error variable σ_ε^2 is required to be constant. When this requirement is violated, then the condition is called heteroscedasticity.

production function is usually fitted as a linear model trough logarithmic transformation and then applying method of least square. He discussed advantage and disadvantages of this function and finally conclude that the assumption of Cobb-Douglas is rarely satisfied in Practice.

3. DATA

Collection of data is the first and very much essential part of analysis. The present study belongs to the estimation of sugar production through Cobb-Douglas model. For this purpose production data of sugar was collected from Matiari Sugar Mill Sindh. Sugar production data is complied yearly and the range of data for present study is of 10 years; from 2000 to 2009 and on the average the every season of sugar is consist of 4 months.

The collected data can be classified in six columns, which are Capital, Labor, Sugar Production, Total Wage, and Wage[4] per Unit (ton) and Sugar Prices per (kg). According to the annual report of Matiari Sugar Mill, the mill has fixed capital in rupees that is RS 65,250,000. For the analysis, we divided total capital into 6525 units and each capital's cost is RS 10,000. Sugar production is measured in Tons and represented in this paper as (production/100). Following graph represents the general behavior of the production data; the graph clearly shows the upward trend and positive relationship. Depiction of **(Fig. 1)** shows sugar production is mostly depends on the labor rather than capital because capital has minor and fixed affect in production.

Matiari Sugar Mill Production (*2000-2009*)

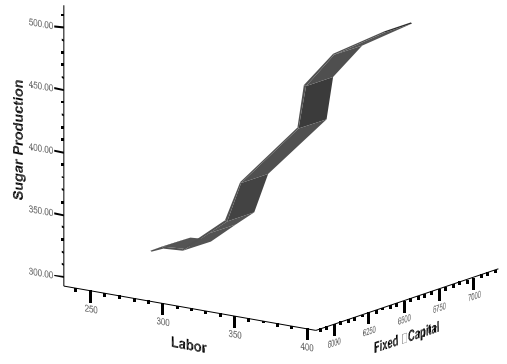


Fig. 1 Sugar Production of Matiari Sugar Mill

$$Average\ production\ of\ a\ labor = \frac{sum\ of\ production\ of\ 10\ years}{total\ number\ of\ labor\ in\ 10\ years}$$

then,

$$wage\ per\ ton = \frac{total\ wage}{average\ product\ of\ a\ labor}$$

4. METHODOLOGY FOR PRODUTION ESTIMATION

i. Cobb-Douglas Production Function

Relationship between the production and factors of production can be expressed through Cobb-Douglas model. It was proposed by Knut Wicksell (1851-1920) and reviewed by Charles Cobb and Paul Douglas in 1928. Cobb and Douglas published a study in which they modeled growth of the American economy. They considered a simplified view of the economy in which production output is determined by the amount of labor involved and amount of capital invested.

$$Q = AK^{\alpha}L^{\beta} \qquad (4.1.1)$$

- Q = Total Production.
- A = Total Factor Productivity.
- L = Labor Input.
- K= Capital Input
- α *and* β output elasticities of Labor and Capital respectively and also measure the contribution of Labor and Capital.

a. Returns to scale

It refers to a technical property of production that examines changes in output subsequent to a proportional change in all inputs. If output increases by that same proportional changes then there are constant returns to scale (CRTS) and sometimes referred to simply as returns to scale. If output increases by less than that proportional change, there are decreasing returns to scale (DRS). If output increases by more than that proportion, there are increasing returns to scale (IRS).

Constant Return to Scale (CRTS).	$\alpha + \beta = 1$
Increasing Return to Scale (IRS).	$\alpha + \beta > 1$
Decreasing Return to Scale (DRS).	$\alpha + \beta < 1$

b. Coefficient of Determination (R-square)

The coefficient of determination is defined as the square of the coefficient of multiple correlation and is denoted by $R^2$and when it is multiplied by 100 it gives the percentage of variance in *dependent* variable which is associated with the variance in *independent* variables. The range of R^2 is between -1 to 1. If the value of R^2 is close to 1 it shows positive relationship and imply that more of the variability in *dependent* variable is explained by the regression model. The sample coefficient of determination is given by:

$$R^2 = 1 - \frac{SS(Residual)}{SS(Corrected)}. \qquad (4.2.1)$$

It is a most common measure of how well a regression model fits the data. Therefore, it is a very necessary statistic to compute for determination of validity of regression model.

c. Standard Deviation of Regression

Standard deviation of regression is always computed to diagnose correct regression model. It is also called standard error of estimates. Here we estimate three coefficients A, α and β and, therefore, the degree of freedom (*n* – *3*) is used for this statistic. It is simply the standard deviation of *dependent* variable's values about the estimated regression and is often used as summary measure of the "goodness of fit" of the estimated regression line. The sample standard deviation of regression denoted by s_{reg} and is given by:

$$s_{reg} = \sqrt{\frac{\sum u_i^2}{n-3}} \qquad (4.3.1)$$

where $\sum u_i^2 = \sum (Q_i - \hat{Q}_i)^2$. (4.3.2)

d. Optimal employment of Labor

Every production estimation problem cannot be solved without considering the labor input rate. In our present study, due to the fixed capital the production problem is heavily dependent upon labor employment. Production manager and analyst want to employ an optimal amount of labor. For this purpose, the basic principle is that additional units of the variable output should be hired until the marginal[5] revenue product (MRP) of the last unit employed is equal to the cost of the input. In general, marginal revenue product is equal to marginal product is multiplied by constant price (P):

$$MRP_L = P.MP_L \qquad (4.4.1)$$

Thus labor is hired until MRP_L equals the wage rate (w): $MRP_L = w$ (4.4.2)

From equation (4.4.1) and (4.4.2) we form:

$w = P.MP_L$ (4.4.3)

where $MP_L = \frac{d}{dL}Q = \frac{d}{dL} AK^{\alpha}L^{\beta}$ (4.4.4)

Hence the optimal labor can be found as:

$w = P.\frac{d}{dL} AK^{\alpha}L^{\beta}$ (4.4.5)

e. SPSS

SPSS (originally, Statistical Package for the Social Sciences) was released in its first version in 1968 after being developed by Norman H. Nie and C. Hadlai Hull. SPSS is among the most widely used programs for statistical analysis in social sciences. Many statistical functions can be performed through this software but in present study we are interested to olve Cobb-Douglas function from *non-linear regression menu*.

Non-linear regression is a method of finding a non-linear relationship between the dependent variable and a set of independent variables. In this paper Sugar Production (Q) is used as a dependent variable, while other two variables Labor (L) and Capital (K) are used as independent variables. Non-linear model building is accomplished by using iterative estimation algorithm and iterations play very important role in finding the optimal values of coefficients. In the present study, the model building of Cobb-Douglas uses three iterations. Besides the iterations, the choice of good starting values is also very important and in this study, according to the assumptions of basic Cobb-Douglas model the values of α and β both use 0.5, while *A* uses 100 because of three digits data (e.g. 604). Two constraints $\alpha \geq 0$ and $\beta \geq 0$ are the essential part of the model which shows the coefficient α and β cannot be negative.

5. RESULTS AND DISCUSSION

The general behavior of the production data in fig. 1 clearly shows upward slope and also provides evidence that the production significantly depends on labor skills. For this purpose the model building of the Cobb-Douglas function between production and factors of production (i.e. capital and labor) is performed by SPSS and estimated coefficients present in **(table 5.1)**.

Table: 5.1 Estimated Parameters of Cobb-Douglas Production Function

Parameter	Estimate	Std. Error
A	.271	3304377.416
alpha	.453	1390485.671
beta	.576	0.265

Hence the estimated Cobb-Douglas production model for Matiari Sugar mill from 2000 to 2009 is:

$$Q = 0.271K^{0.453}L^{0.576} \qquad (5.1)$$

As we discussed in section (4.1.1), return to scale shows the elasticities of labor and capital with production, which is calculated by summing of exponent coefficients[6], $\alpha + \beta = 1.029$. The above calculated value is greater than one and hence the present production data has increasing return to scale.

Two diagnostic statistics *R-square* and *standard deviation of regression* are used for the correct fitting of regression model. The ANOVA table 5.2 presents sum of squares and mean squares. The value of R^2 is 0.724, which shows the adequacy of model and standard error of estimate with *n – 3* degrees of freedom to be 46.35 tons.

Optimal employment is in the Appendix of this paper which was discussed in Section 4.4 to find

[5] **Marginal means, the change in dependent variable associated with a one-unit change in independent variable. Marginal function is usually dealt as derivative.**

[6] **The coefficients of Cobb-Douglas function α and β are placed in a function as an exponent.**

Table: 5.2 ANOVA

Source	Sum of Squares	df	Mean Squares
Regression	1530404.731	3	510134.910
Residual	15044.399	7	2149.200
Uncorrected Total	1545449.130	10	
Corrected Total	54523.974	9	

the labor input rate. In this Section, we need wage per unit (ton) of labor and sugar price per kg and MP_L:

$$MP_L = 0.1561 \frac{K^{0.453}}{L^{0.424}} \quad (5.2)$$

The optimal input rates of the labor for year 2010 and 2011 are present in **(table 5.3)**.

Table: 5.3 Estimation table for labor and Production of year 2010-2011

Years	Capital *(units)*	Labor *(units)*	Sugar Production (00) *tons*	Total Wage	Wage / *ton*	Sugar Price RS *(per kg)*
2010	6525	114	252.65	7000	56	50.11
2011	6525	166	274.22	*7000**	*56*	*58.56**

*wage and sugar price for 2011 is to be expected.

The sugar production of 2010 and 2011 is estimated on the basis of estimated labor input while the estimation of labor input depends on wage[7] per ton and sugar price[8] per kg. If wage rate per ton is lower or per kg price of sugar is higher, more labor will be purchased, as we discussed in table 5.3 and it also shows 52 units of labor have increased, which improved production from 252.65 to 274.22 tons.

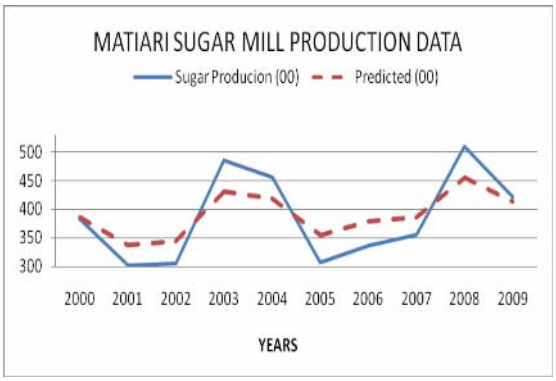


Fig. 2 Original and Predicted Sugar Production of Matiari Sugar Mill

The above diagram shows appropriate fitting of the production of Matiari sugar mill data from 2000-2009.

6. CONCLUSION

This paper has illustrated that Cobb-Douglas function perfectly satisfy the production process and is also useful to determine the optimal input rate. The value of R^2 is 0.724 that shows the goodness of fit of the estimated regression line, because 72% variation in the production is associated with the variation in Labor and Capital. Figure: 2 provide evidence of better fitting of model because the difference between estimated line and original is moderate and predicted line is following to the original line. We conclude that Cobb-Douglas model is a better fit of Matiari Sugar Mill production data and determines optimal labor input rate for 2010 and 2011, which are 114 and 166 respectively.

7. ACKNOWLEDGEMENT

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