Sindh Univ. Res. Jour. (Sci. Ser.) Vol.43 (2) 237-242 (2011)

SINDH UNIVERSITY RESEARCH JOURNAL (SCIENCE SERIES)

Finite element simulation of Steady Newtonian Blood flow in Carotid artery bifurcation

UBAIDULLAH* AND M. S. CHANDIO

Institute of Mathematics and Computer Science, University of Sindh, Jamshoro, Pakistan.
Corresponding author: M. S. CHANDIO, E-mail: mschandio@hotmail.com Cell. No. 92-3322060869

Received 08rth January 2011and Revised 11th August 2011)

Abstract: Blood flow simulation in carotid artery bifurcation is presented here. Newtonian flow is considered for the modeling of carotid artery flow using Finite element method. Fluid-structure interaction between the wall and blood is neglected and rigid carotid artery is considered here. Simulation is performed at various Reynolds numbers on both normal and stenotic carotid arteries. Parameters of interests, like, shear-stress and velocity are obtained in detail through a stenotic carotid artery bifurcation for the obstruction ratio, ranging from 30% to 80%. For the purpose of numerical simulation, Semi-implicit Taylor-Glarkin Pressure-Correction Finite element Method (Townsend and Webster, 1987) is employed here.

Keywords: finite element, Blood flow simulation, Newtonian flow, carotid artery

1. INTRODUCTION

The flow of Blood in human arteries has been an area of current research. The study is so important because the majority of the people of developed countries heart attacks that subsequently cause heart failure. The Common Carotid Artery transports blood to head, neck and brain. CCA is further divided into Internal Carotid Artery (ICA) and External Carotid Artery (ECA). ICA supplies blood to brain, which is taken as the domain of our study.

Atherosclerosis is a disease which deposits plaque inside the arterial walls that narrows the passage area and prevents the smooth flow of blood in the arteries. Sometimes these small pieces of plaque dislodge and converted into a lump and move downstream that further blocks blood vessels. If the opening of the arteries is reduced to a severe level due to the plaque deposition, an individual may suffer from brain hemorrhage (Stroke) or cardiac failure that may cause sudden death. Literature survey has revealed that the wall shear stress is most concerned to the high and low shear regions. Generally Blood is considered as Newtonian fluid, particularly in arteries like carotid. Numerous experimental and numerical techniques have been used by many scholars to investigate the formation of plaque and its size. These scholars in their research work have also computed shear stress at various Reynolds numbers with different obstruction ratios in the diseased carotid arteries. Below is given brief literature review.Baghdadi (2002) and Salzar and *et al*., (1995) have reported that millions of people worldwide are affected by the atherosclerosis that leads to diseases as heart failure, brain stroke and long-term disability. Deshpande *et al*., (2009) simulated Pulsatile blood flow in carotid artery bifurcation using incompressible Navier–Stokes equations. Deshpande *et al*., (2009) further quoting WHO reports (2006,2008), they mentioned that around 13 percent deaths, *i.e.* around 18 million people, died in 2005 due to cardiac problems worldwide. About 80 percent of these deaths took place in developing or under developing countries. According to WHO reports (2066,2008), if this trend is continued, the number of deaths due to these diseases *i.e.* heart failures and strokes, may rise to 20 million by 2015. Boyd *et al*., (2006) used a 2D carotid artery model employing Lattice Boltzmann method to investigate the validity of Newtonian viscosity near wall shear stress in a two-dimensional carotid artery model using the Lattice Boltzmann Method using Carreau-Yasuda blood viscosity model. These authors in their previous work have shown that the assumption of Newtonian viscosity may not be suitable for brachial arteries (Boyd *et al.*, (2006)). Zarins *et al.*, (1983) and Glagov *et al.*, (1988) in their separate works observed that the flow disturbances play a major influence in the localization of the disease. Bharadvaj *et al.* (1982) has made a significant research work. They have analyzed the hemodynamic flow patterns in the carotid artery. Giddens *et al*., (1993) have reported that Atherosclerosis and region of abnormal wall shear stress in arteries are interrelated. Golpayeghani *et al*., (2008) in their numerical work have used incompressible Navier-Stokes equations. They studied the effects of local stenosis in carotid artery. The parameters of interest they focused upon were the time-

*Department of Mathematics, Karakoram International University of Gilgit, Gilgit-Baltistan, Pakistan

dependent velocity profile in various regions and flow separation and reattachment points on the wall. They also investigated wall shear stress variation in whole artery and compared the results with non-Newtonian fluid. Measurement of turbulent flow fields are reported by Deshpande and Giddens (1980). Berger and Jour. (2000) made a comprehensive study of the stenotic flow. Senol *et al.*, (2010) using 3D unsteady Navior-Stokes equation simulated the biomechanical properties based on real carotid artery. In their studies they used different inlet velocity profiles. They gathered data using CT of a real patient for the computational domain. They observed that the velocity, pressure and wall shear stress are greatly affected by input boundary conditions. Lee *et al.* (2008), Zhao *et al.*, (2000), Starmans-Kool *et al.*, (2002) and Wake (2005) in their works obtained data using MRI, MRA and pulsed Doppler Ultra Sound. Mickaël Gay and Lucy T. Zhang (2008) performed numerical analysis of blood flow to identify the causes of onset, growth and possible factors that leads to severe obstruction in the passage of diseased carotid artery. Baiijens *et al.* (1993) has simulated Newtonian and Generalized Newtonian blood flow using 2D carotid artery bifurcation model.

Governing Equations

The flow of blood in carotid artery is considered as steady, incompressible, laminar and two-dimensional. In this paper, the blood is considered to be Newtonian. The flow is considered as isothermal due to effect of temperature change is very low (Senol *et al.*, 2010). The governing equations comprise Momentum and continuity equations. These may be described as:

$$\rho \frac{\partial u}{\partial t} = \nabla .T - \rho u \nabla u - \nabla p \qquad (1)$$

$$\nabla .u = 0 \qquad (2)$$

where t is time, u(x,t) is fluid velocity, x is space and p is isotropic pressure variables. Material density is ρ. For Newtonian flows, the stress $\boldsymbol{T}$ is defined via a Newtonian viscosity μ_0, and the rate-of-deformation tensor $\boldsymbol{D}$, and velocity gradient L, as

$$\boldsymbol{T} = 2\mu_0 \boldsymbol{D} \qquad (3)$$

Where

$$\mathbf{D} = \frac{L + L^t}{2} \quad \text{and} \quad L^t = \nabla u \cdot$$

Employing continuity equation (2) with constant viscosity, famous Navier-stokes equation is obtained as follow:

$$\rho \frac{\partial u}{\partial t} = \mu_0 \nabla^2 u - \rho u \nabla u - \nabla p \qquad (4)$$

Characterizing non-dimensional variables u = Uu* and p = [μU/L]p*, discarding the * notation, we obtained an equivalent non-dimensional system of equations to (2) and (4) as:

$$\mathrm{Re} \frac{\partial u}{\partial t} = \nabla^2 u - \mathrm{Re}\, u \nabla u - \nabla p \qquad (5)$$

$$\nabla .u = 0 \qquad (6)$$

where the non-dimensional Reynolds number

$$\mathrm{Re} = \frac{\rho UL}{\mu} .$$

Numerical Method

In this work, Finite element scheme is used. The general framework is based upon semi-implicit Taylor-Galerkin/Pressure-correction(SITpak) algorithm as proposed by Townsend and Webster (1987). The accuracy, stability and consistency of this algorithm have been demonstrated for both closed domain and free surface complex problems. Aboubacar and Webster (2001) used this algorithm for contraction problems, Chandio *et al.*, (2004, 2002) employed this algorithm for free-surface filament stretching and reverse roller coating problems and V. Ngamaramvaranggul and Webster (2002) used this scheme for wire-coating problem. The details upon SITpak algorithm may be found in Chandio and Webster (2002). Using the notations of Cuvelier *et al.*, (1986), the velocity filed is approximated by $U(\mathbf{x},t)=U_j(t)\phi_j(\mathbf{x})$ and pressure field by and $P(\mathbf{x},t) = P_k(t)\psi_k(\mathbf{x})$. Here, ϕ_j is piecewise quadratic and ψ_k linear basis functions. The fully-discrete semi-implicit Taylor-Galerkin/pressure-correction system of equations may be expressed in matrix form as (Chandio *et al.*, 2002):

Stage1a:

$$\left(\frac{2\mathrm{Re}}{\Delta t} M + \frac{1}{2} S\right)\left(U^{n+\frac{1}{2}} + U^n\right) = \left\{-\left[S + \mathrm{Re}\, N(U)\right]U + L^T P\right\}^n$$

Stage 1b:

$$\left(\frac{\mathrm{Re}}{\Delta t} M + \frac{1}{2} S\right)\left(U^* - U^n\right) = -\left[SU + L^T P\right]^n + \left[\mathrm{Re}\, N(U)U\right]^{n+\frac{1}{2}}$$

Stage 2: $K\left(P^{n+1} - P^n\right) = -\frac{2}{\Delta t} LU^*$

Stage 3: $\frac{1}{\Delta t} M\left(U^{n+1} - U^*\right) = \frac{1}{2} L^t \left(P^{n+1} - P^n\right)$

where M represents mass, S diffusive, N(U) convection, L pressure gradient, and K pressure stiffness matrices. The detailed summary about the indicial notations of above matrices are given in Chandio and Webster (2002).

Boundary Conditions

The geometry in the current study is based on the dimensions and work of Bharadvaj *et al.*, (1982),

Perktold *et al*., (1991) and Baaijens *et al*. (1993). The schematic diagram of the problem geometry is shown in **(Fig.1)**. Here, the walls of the carotid artery bifurcation are taken as rigid-walls. Fully developed flow is considered at the inlet. No slip boundary conditions are imposed on the walls. Pressure at the outlet is set to zero.

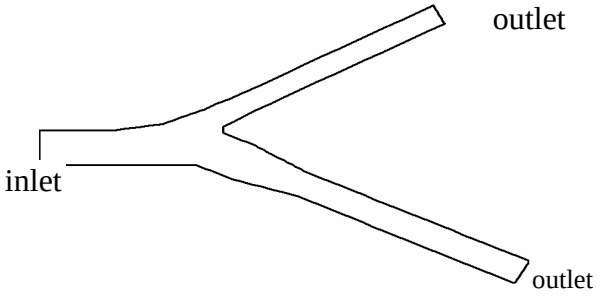


Fig.1: Schematic diagram of carotid artery bifurcation

2. RESULTS AND DISCUSSIONS

In this paper, solution is obtained at Reynolds numbers of 10, 100, 400, 800 and 1000 with increasing fluid velocity for different percentage of stenosis ranging from 30% to 80%. The results are obtained in Cartesian frame of reference and are given in non-dimensional form for convenience of demonstration. A single mesh is used for the purpose of simulation. A slight modification in the mesh is made at the entrance of ICA to reduce the ICA diameter by 30%, 50% and 80%. The meshes are shown in figure 2 below. Mesh for normal carotid artery and meshes with 30% and 50% stenosis consist of 1492 elements and 3159 velocity nodes and 834 pressure nodes. Mesh with 80% stenosis is refined near the occlusion region for getting better accuracy due to highly curved region. That mesh consists of 1602 elements, 3389 nodes and 894 pressure nodes. In the current study, the blood is considered to be Newtonian, homogeneous, isothermal and an incompressible fluid with a constant dynamic viscosity of 0.0035 Pa. and density 1050 kg/m^3. (Yoshiyuki *et al*., 2008). A typical steady-state flow pattern in normal carotid artery is depicted in **(Fig. 3)** below, which shows velocity contours. A fully developed profile is obtained at the inlet and so at the outlet regions. In this study, we have focused upon shear rates and onset of vortex development in and around the stenosis region. Velocity vectors in the normal carotid artery show that the flow is smooth and parabolic in the entire carotid artery see **(Fig. 4a)**. However, for stenosed arteries, a disturbance is observed near the apex (point of division of CCA into ICA and ECA) where the fluid is entering the ICA with accelerated speed, see **(Fig. 4 b,c,d).** It is also seen around the stenosis near the lower wall of the ICA, see Fig. 4 (c,d). This indicates the onset of vortex due to flow circulation in that region, see **(Fig. 5 c,d)**. The onset of vortices around the stenosis indicates the further development of plaque downstream the ICA, which may cause multiple stenosis in the carotid artery bifurcation.

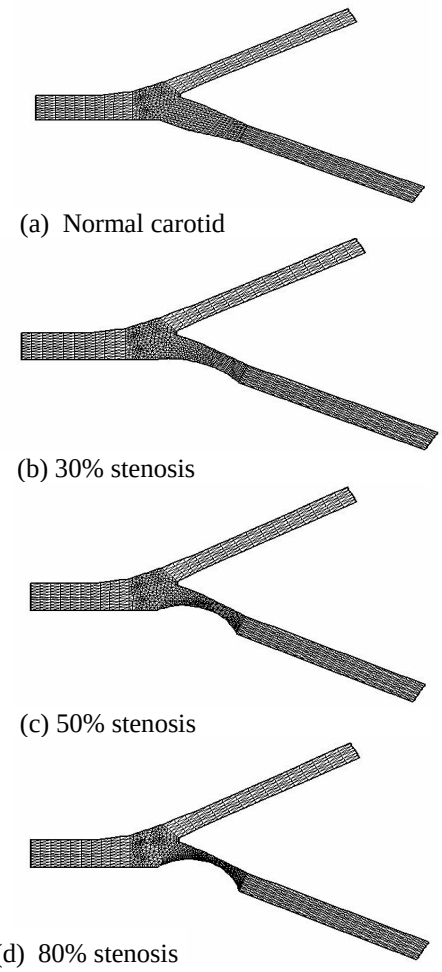


Fig. 2. Meshes of Carotid Artery Bifurcation at various percentage of stenosis.

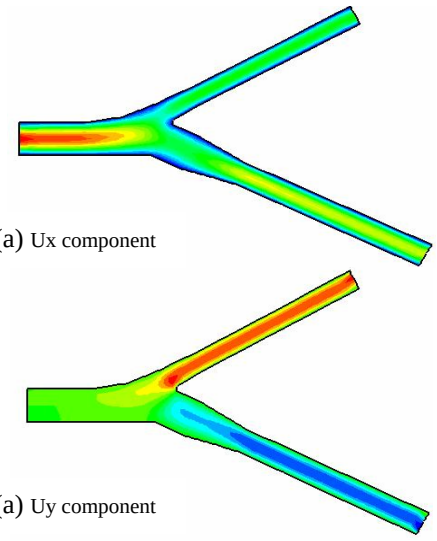
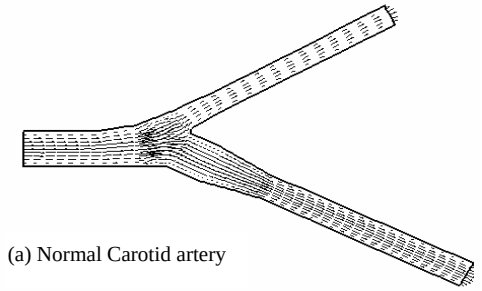


Fig. 3: Velocity distribution in the Normal carotid artery

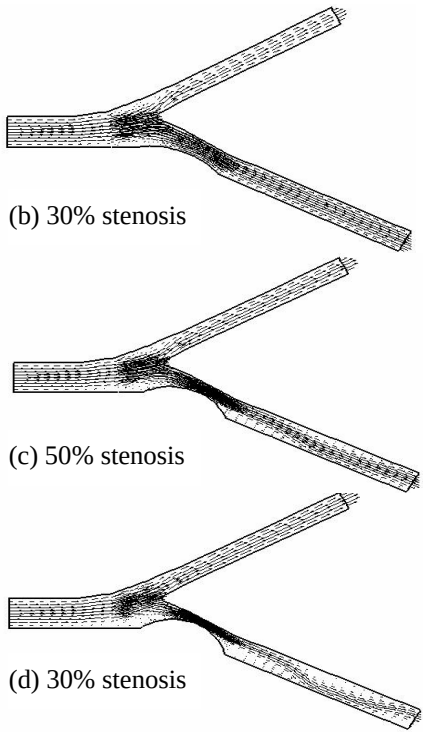
A comparative study based on shear rates across various Reynolds numbers and percentage of stenosis is made. We know that large stresses are created due to high shear rates. In our study, we found that high shear rates are created around the region of stenosis. Higher the percentage of stenosis larger the shear rates observed with increasing Reynolds number, see **(Fig. 6 and 7)**. We observed that for Re=10 the maximum shear rate of about 7 units is obtained for 80% stenosis, see **(Fig. 6a)**. This trend has continued and maximum shear rate of about 17 unit is achieved for Re=1000 for 80% stenosis, see **(Fig. 6 b,c,d,e)**. Similar trend is observed across various percentage of stenosis, i.e. maximum shear rate is obtained for 80% stenosis at Re=1000, see **(Fig. 7)**. Thus high shear rates are caused due to high velocity gradient in the narrowed region.

4. CONCLUSION:

Finite element simulation of carotid artery bifurcation is performed using SITpak algorithm. In this paper, results are obtained for diseased arteries having 30%, 50% and 80% obstruction in the ICA. Various Reynolds numbers are used that indicates the fluid flow speed in the domain. We found that the flow in the normal carotid artery is smooth and produces very low shear rates of units less than 3 for all Reynolds numbers, see **(Fig. 7a)**. However, with increasing Reynolds number as well as percentage of stenosis these shear rates reached to about 17 units, which are 6 times larger than that of normal carotid artery. We also observed the onset of vortices at 50% obstruction which indicates that the occlusion above 50% requires immediate attention. These vortices before the stenosis may cause recirculation in the blood that may produce high stresses in that region and subsequently cause heart attack or stroke. The presence of these vortices around diseasesd region is an alarming situation. These vortices, with the lapse of time, may build up plaque near the stenosis region and develop multiple stenoses in the downstream of ICA. We also proved that the SITpak algorithm is consistent and work well with blood flow simulation problems as well. The future study will be devoted to generalized Newtonian flow.



(a) Normal Carotid artery

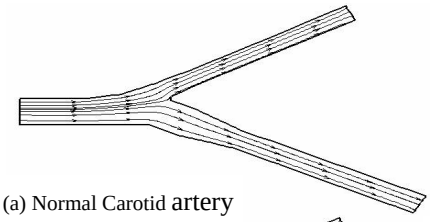


(b) 30% stenosis

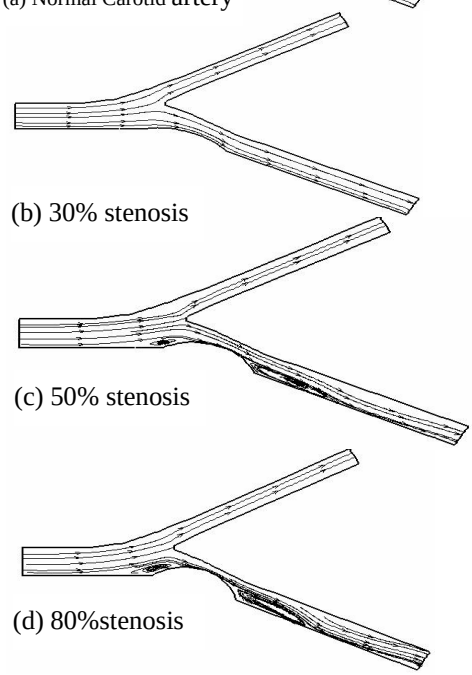
(c) 50% stenosis

(d) 30% stenosis

Fig. 4: Velocity vectors at different percentage of occlusions at Re=800



(a) Normal Carotid artery

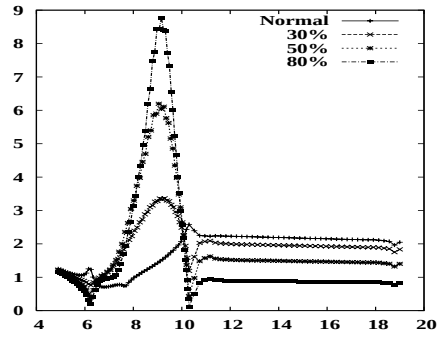


(b) 30% stenosis

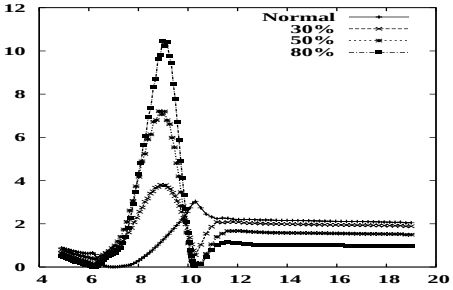
(c) 50% stenosis

(d) 80%stenosis

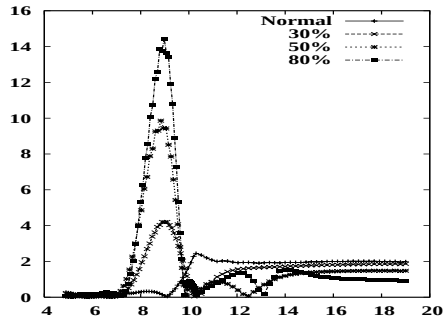
Fig. 5: streamline patterns at different percentage of occlusions at Re=800



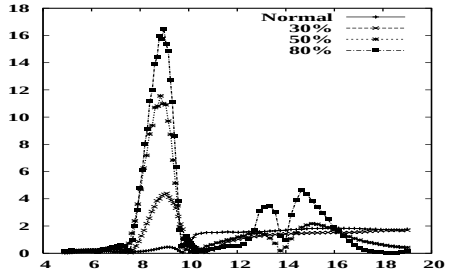
(a) Re=10



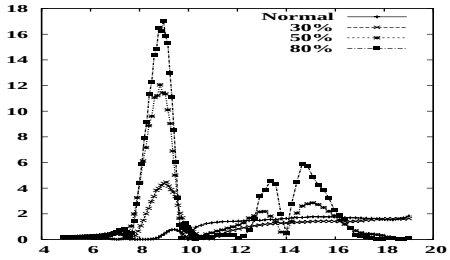
(b) Re=100



(c) Re=400

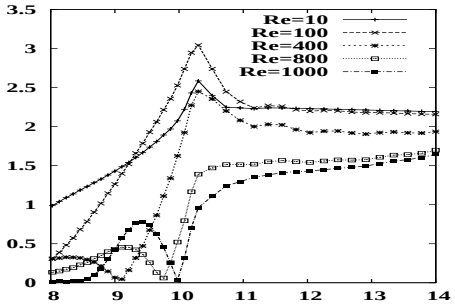


(d) Re=800

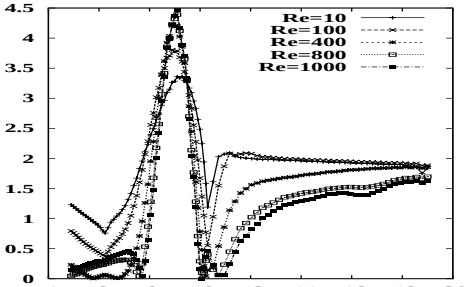


(e) Re=1000

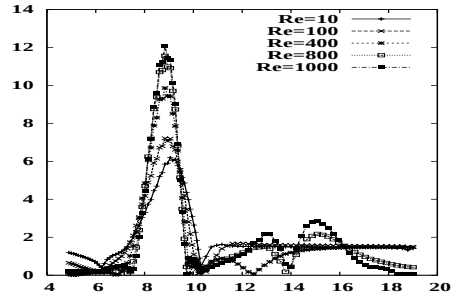
Fig. 6: shear rate distribution along the lower wall of ICA across different Reynolds numbers for various percentages of occlusions.



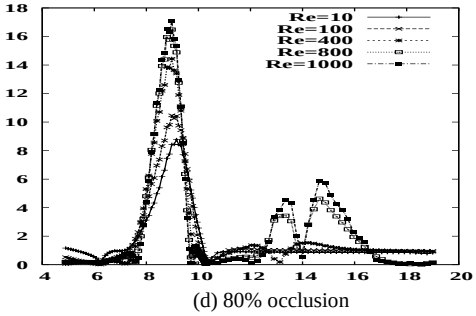
(a) Normal carotid artery



(b) 30% occlusions



(c) 50% occlusion



(d) 80% occlusion

Fig. 7: shear rate distribution along the lower wall of ICA across for various percentages of occlusions at different Reynolds numbers.

REFERENCES:

Aboubacar M. and M.F. Webster (2001) A cell-vertex finite volume/element method on triangles for abrupt contraction viscoelastic flows, J. Non-Newt. Fluid Mech., vol. 98 (**2**): 83-106.

Baghdad L. (2002) Template-Based Finite Element Mesh Generation From Medical Images, Department of Medical Biophysics, M.Sc. Thesis, The University of Western Ontario London, Ontario, Canada.

Boyd J., J.M. Buick and S. Green (2006) Application of the Lattice Boltzmann Method to non-Newtonian flow in a carotid artery model, Australian Institute of Physics 17th National Congress 2006–Brisbane.

Boyd J., J.M. Buick and S. Green (2006) Comparison of Newtonian and Non-Newtonian Oscillatory Flows Using the Lattice Boltzmann Method. In Cardiac Mechanics and Hemodynamics, Proceedings of the World Congress on Medical Physics and Biomedical Engineering. Seoul.

Berger, S.A. and L.D. Jou, (2000) Flows in stenotic vessels. Annual Rev. Fluid Mech., vol. (**32**): 347–382.

Baaijens, J.P.W., A.A. Steenhoven, and J.D. Janssen, (1993) Numerical analysis of steady generalized Newtonian blood flow in a 2D model of the carotid artery bifurcation. Biorheology, vol. 30 (**1**): 63-74.

Bharadvaj, B. K., R. F. Mabon, and D. P. Giddens, (1982),''Steady Flow in a Model of the Human Carotid Bifurcation. Part 1 Flow Visualization,'' J. Biomech., vol. (**15**): 349–362.

Cuvelier C., A. Segal, and A.A.V. Steenhoven (1986) Finite Element Methods and Navier-Stokes Equations, D. Reidel Publishing Company.

Chandio M.S., K. Sujatha and M.F. Webster (2004) Consistent hybrid finite volume/element formulations: model and complex viscoelastic flows, Int. J. for Numerical Methods in Fluids, vol. 45 (**9**): 945-971.

Chandio M. S. and M. F. Webster (2002) Numerical Simulation for Viscous Free-surface Flows for Reverse Roller-Coating, Int. J. Num. Meth. Heat Fluid Flow, vol. (**4**): 434-457.

Deshpande1 M.D., V. Ballal1, S.R. Shankapal1, and M.G. Srinath (2009) Subject-specific blood flow simulation in the human carotid artery bifurcation Current Science, vol. (**97**): 9, 1303-1312.

Deshpande, M.D. and D.P. Giddens (1980) Turbulence measurements in a constricted tube. J. Fluid Mech., vol. (**97**): 65–89.

Glagov, S., C. Zarins, D.P. Giddens, D.N. Ku, (1988) Hemodynamics and atherosclerosis: Insights and perspectives gained from studies of human arteries. Arch Pathol Lab Med, vol. (**112**): 1018-1031.

Giddens, D.P., Zarins, C.K. and Glagov, S.J. (1993) The role of fluid mechanics in the localization and detection of atherosclerosis, J. Biomech. Engg. vol. 115 (**4B**): 588-594.

Golpayeghani A. T., S. Najarian, M.M Movahedi (2008), Numerical Simulation of Pulsatile flow with Newtonian and non-Newtonian behavior in arterial stenosis, Iranian Cardiovascular Research Journal vol. (**1**): 167-174.

Gay M. and L.T. Zhang (2008) Numerical studies of blood flow in healthy, stenosed, and stented carotid arteries, Int. J. No. Meth. Fluids, vol. (**61**) 4, 453-472.

Lee, S.E., S. Lee, P. F. Fischer, and F. Loth, (2008) Direct numerical simulation of flow in a stenosed carotid bifurcation. J. Biomech., vol. (**41**): 2551 2561.

Piskin S., E. Aribas and M. S. Celebi (2010) A 3D Human Carotid Artery simulatiom USING Realistic Gemoetry and Experimental Input Data, V European Conference on Computational Fluid Dynamics ECCOMAS CFD 2010 J. C. F. Pereira and A. Sequeira (Eds) Lisbon, Portugal.

Starmans-Kool, M.J., A.V. Stanton, S. Zhao, X.Y. Xu, Thom, S.A.M. and A.D. Hughes (2002) Measurement of Hemodynamics in human carotid artery using ultrasound and computational fluid dynamics. J. Appl. Physiol., vol. (**92**): 957–961.

Townsend P. and M. F. Webster (1987) An algorithm for the three-dimensional transient simulation of non-Newtonian fluid flows, Proc. NUMETA 87, Martinus Nijhoff, Publishers, Dordrecht, 2: T12/1.

Perktold, K., Resch, M. and Florian, H. (1991) Pulsatile Non-Newtonian Flow Characteristics in a Three-Dimensional Human Carotid Bifurcation Model, ASME J. Biomech. Eng., vol. (**113**): 464–475.

Maruyama((2008), Three-dimensional numerical simulation of blood flow in the aortic arch during cardiopulmonary bypass, European Journal of Cardio-thoracic Surgery, vol. (**33**):164-167.

Ngamaramvaranggul, V. and M. F. Webster, (2002), Simulation of pressure-tooling wire-coating flow with Phan-Thien/Tanner models, Int. J. Num. Methods in Fluids, Vol. 38 (**7**): 677-710.

Salzar, R. S., M. J. Thubrika, and R. T. Eppink (1995) Pressure-induced mechanical stress in the carotid artery bifurcation: A possible correlation to atherosclerosis. Journal of Biomechanics, vol.28 (**11**):1333-1340.

Wake, A. K. (2005) Modeling Fluid Mechanics in Individual Human, Carotid Arteries, Ph D thesis, Georgia Inst. Tech.

Zarins, C.F., D.P. Giddens, B.K. Bharadvaj, V.S. Sottiurai, R.F. Mabon, (1983) Carotid Bifurcation Atherosclerosis. Circ Res. vol. (**53**): 502-514.

Zhao, S.Z., X.Y. Xu, A.D. Hughes, S.A. Thom, A.V. Stanton, B. Ariff, and Q. Long (2000) Blood flow and vessel mechanics in a physiologically realistic model of a human carotid arterial bifurcation. J. Biomech., vol. (**33**): 975–984.