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A Markov Chain Model For The Probabilities Of Wet And Dry Spells (Case Study For Location Karachi)

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Abstract: Markov Chain model was used to evaluate probabilities of getting a sequence of wet and dry days, which is essential for many practical applications such as to design and predict the yields of different crops. One of the challenging problems for agricultural experts is the random nature of rainfall. The intensity of the rainfall is greatly influenced by the local climatical condition. This model is used to predict the long term weather condition using the previous data 1987-2006 for location Karachi.

Keywords: Rainfall, Relative frequencies, Transitional probabilities, Power of transitional matrix, Markov chain model, Long term prediction.

1. INTRODUCTION

A public interest on climate change has risen sharply in recent years by Dai *et al.,* (1998). Climatically Sindh was divided into three sections, Siro (upper section centered on Jacobabad) , Wicholo (middle section centered on Hyderabad) and Lar (lowest section centered on Karachi).There are so many centers for measuring rainfall in Sindh but this work is selected for location Karachi. This work is related to climate extreme events due to very long sequences of dry spells. A sequence of daily rainfall is essential for many practical applications such as to design and predicting the yields of different crops; yields play an important role in the natural income. For analyzing rainfall data an appropriate approach is to fit the model for the probability of occurrence of rain. Markov chain parameter is used and seasonal variation will be formed with transitional probabilities of daily rainfall data for 1987-2006. By the Kolmogorov equation long term weather will be predicted. The daily rainfall data was collected from Central Data Processing Centre (CDPC) Karachi. Collected data was distributed into four seasons, winter rainfall, Pre monsoon rainfall (Spring), Monsoon rainfall (Summer) and Post monsoon rainfall (Autumn) seasons.

2. MATERIALS AND METHODS

The data is analyzed with season wise distribution of year. Series of D and W were made and converted into tabular form. In this table the first row gives the counts for transition out of dry days i-e dry to dry day and dry to wet day, second row indicates the transition out of wet days i-e wet to dry day and wet to wet day. For analyzing rainfall data an appropriate approach is to fit the model for the probability of occurrence of rain. First order Markov chain parameter was used and seasonal variation was founded by n-step transitional probabilities by Kolmogorov equation i-e:

$$P_{ij}^{(n)} = \sum P_{ik}^{(m)} P_{kj}^{(n-m)} \quad \text{for all i,j and} \quad 0 \le n \ge m$$

Here two state Markov chain was applied regarding one state as "success" i.e. wet day denoted by 1 and other as "failure" i.e. dry day denoted by 0. If the probability of nth trial is failure, the failure probability of the $(n+1)^{th}$ trial is denoted by $1-\alpha$ and the probability of success is α, similarly if the n^{th} trial results in success then the probabilities of (n+1)th trial is denoted by $1-\beta$ and β for success and failure respectively.
Thus two state MC model is

$$P = \begin{pmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{pmatrix}$$

Two state MC model gives a good description of the occurrence of wet and dry days. Estimating the transitional probabilities Pi (t) i-e the probability of

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rain on day t given day t-1 was either dry or wet (i=0,1) can be obtained by using the following relative frequencies. These observed proportions for each day can be used to fit curves for the smoothed estimates of the probability of rain. Let the row vector $P^{(n)} = (P_0^{(n)}, P_1^{(n)})$ denotes the probability of finding the system in state 0 or 1 at time t , when initial probabilities of two states are given by $P^{(0)} = (P_0^{(0)}, P_1^{(0)})$. Taking the event of being in state 0 in time t, this event can occur in two mutually exclusive ways, either state 0 was occupied at time t-1 and no transition out of state occurred at time t is probability $P_0^{(n-1)}(1- \alpha)$ or state 1 is occupied at time t-1 and a transition from state 1 to state 0 occurred at time t is probability $P_1^{(n-1)}(\beta)$, having the following relation

$P_0^{(n)} = P_0^{(n-1)}(1- \alpha) + P_1^{(n-1)}(\beta)$

$P_1^{(n)} = P_0^{(n-1)}(\alpha) + P_1^{(n-1)}(\mathbf{1}-\beta)$

In matrix notation $P^n = P^{n-1}.P$, now we will find the two step, three step and n step transitional matrices. If the initial state of the system is 0 then $P^{(0)} = (1,0)$. This represents a vector in which dry day is 100% and wet day is 0%. The ij^{th} entry in P^n is the probability that the system will pass from state i to state j in n steps. After sufficient long period of time the systems settle down to a condition of statistical equilibrium. The steady state distribution vector may be founded by solving the equation XP = X, i-e long term prediction. In last we will find the expected number of wet and dry days by the following relation $E(W) = \sum n (1-\beta)^{-1} \beta = (\beta)^{-1}$ and similarly $E(D) = \sum n (1- \alpha)^{-1} \alpha = (\alpha)^{-1}$

3. Literature Review

Stem RD and Coe, (1984) gave a way to analyze and simulate the daily rainfall probabilities and probability of number of rainy days. Richard W. Katz, (1974) used Markov Chain to find probabilities of rainfall data. Emily S Murphy, (1995) has formed Markov chain model and probability distribution. Faqeer Muhammad, (1991) has formed Markov chain model for rainfall data in Pakistan. Zahid Hussain, (2001) formed Markov chain to daily rainfall data using Fourier series. E.Di Giuseppe, (2005) analyzed wet and dry spells. Mohammad Muslehuddin, (2005), Precipitation plays an important role in social and economic development of Pakistan. Avik Ghosh, (2010), two state Markov chain was applied four stations of Genetic West Bengal. A.K. Sahai, (2003), Markov chain models are very significant, indicated that 80% of rain received in only Monsoon season. In this study we deal with the waiting time problems for the first consecutive success of a specified length and for sooner and later runs between success run and failure run with specified length respectively of the rainfall data for last 20 years of the location Karachi by Markov Chain for two states.

4. Data and Analysis

Daily and monthly rainfall data for period 20 years1987 to 2006 for Karachi was used in the present study. Data was collected from Central Data Processing Centre Met Complex Karachi. In this study data was distributed into four seasons, winter rainfall season (January to March), Pre Monsoon rainfall season (April to June), Monsoon Rainfall season (July to September), Post Monsoon rainfall season (October to December). Firstly transitional probabilities are formed and Markov chain model is fitted to each season.

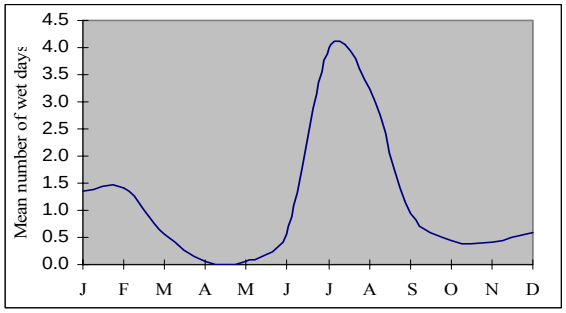

Fig.1 The probability of wet days month wise.

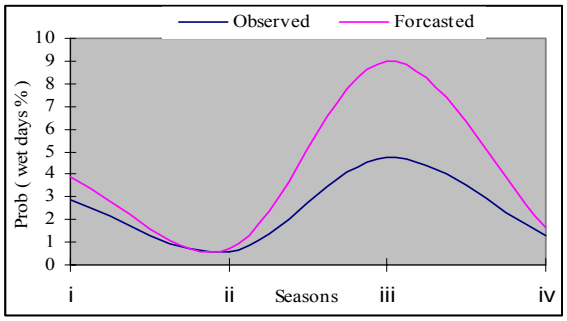

Fig.2 Comparison of observed and Forecaste Probability of wet days by season wise.

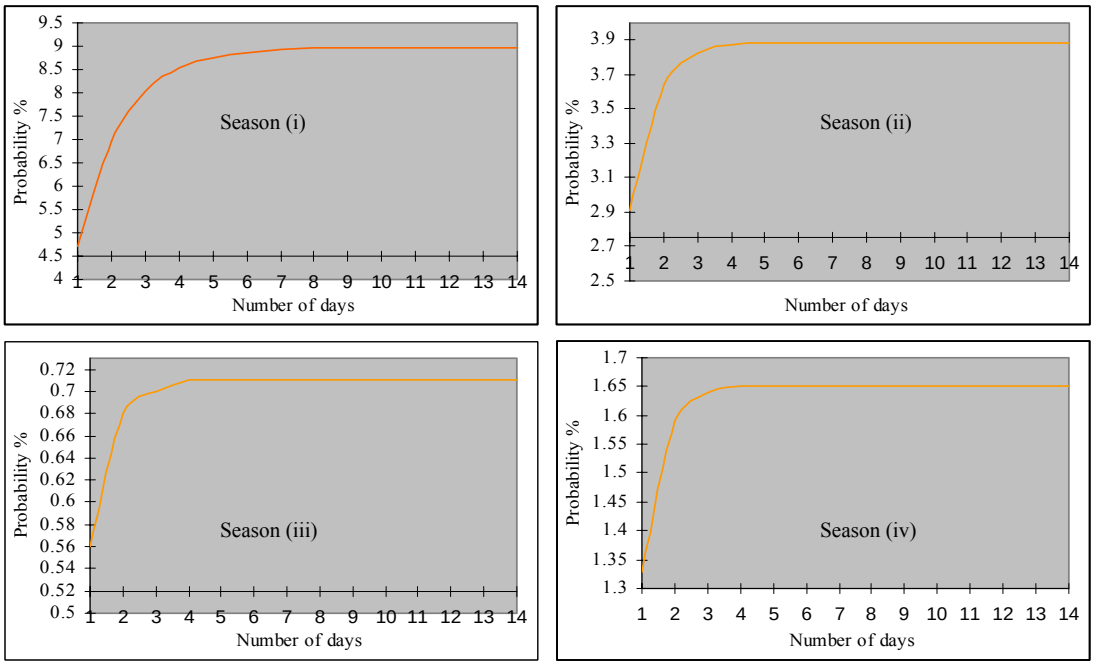

Fig. 3-6 The trend line of days required to reach at equilibrium condition by season wise.

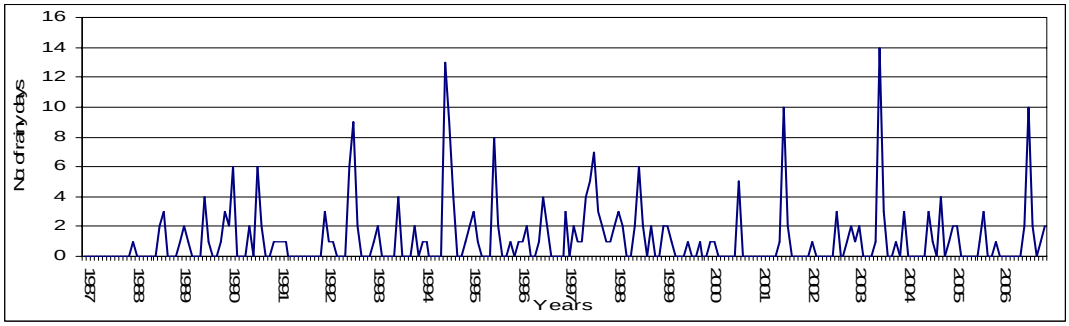

Fig. 7 Monthly number of wet days in different years over Karachi during 1987-2006

5. RESULTS

Using the Markov chain model the results were obtained. In winter Rainfall season the transition probability matrix P indicates that 97.09% chance of dry days, the probability of dry days is 0.9709. Expected combined spell of wet and dry days is 36 days. It was forecasted that in the long term 96.11% of days will be dry days. In Pre Monsoon rainfall season the transition probability matrix P indicates that 99.44% chance of dry days, the probability of dry days is 0.9944. Expected combined spell of wet and dry days is 180 days. It was forecasted that in the long term 99.30% of days will be dry days. In Monsoon rainfall season the transition probability matrix P indicates that 95.28% chance of dry days, the probability of dry days is 0.9528. Expected combined spell of wet and dry days is 24 days. It was forecasted that in the long term 91.03% of days will be dry days. In Post Monsoon rainfall season the transition probability matrix P indicates that 98.67% chance of dry days, the probability of dry days is 0.9867. Expected combined spell of wet and dry days is 78 days. It was forecasted that in the long term 98.36% of days will be dry days.

Table.1 Transitional probability matrices for daily precipitation for four seasons from 19887-2006.

Winter rainfall			Pre Monsoon			Monsoon			Post Monsoon		
	D	W		D	W		D	W		D	W
D	0.9709	0.0291	D	0.9944	0.0055	D	0.9528	0.0472	D	0.9867	0.0133
W	0.7206	0.2784	W	0.7857	0.2143	W	0.4790	0.5210	W	0.7931	0.2069

Table.2 Steady state distribution probability vector, Expected length of different spells, Weather cycle and number of days for equilibrium state.

Seasons	X_1	X_2	Expected length of			N	Total days
			Dry	Wet	Cycle		
Winter rainfall	0.9611	0.0389	34	2	36	7	90
Pre Monsoon	0.9930	0.0070	179	1	180	7	91
Monsoon	0.9103	0.0897	21	2	23	12	92
Post Monsoon	0.9836	0.0164	76	1	77	7	92

6. CONCLUSION

Rainfall is the critical factor in farm management. The MCM, the rain prediction model based on data for 20 years, (1987-2006).This study carried out to access the MCM initialized for forecasting rainfall from data proceeded rain ratios in location Karachi. Month wise, July is more rain producing month than others months produces 20-30% rain of whole year. The MCM conclude that there is decrease in dry days than wet days. This results a prediction of slow increase of rains may be forecasted in coming weather situations. From this analysis it was observed that the climate scene regarding the particular aspects of precipitation has changed and there was a longer period of absence of rainfall found. This model carries a strategy to manage crops according to weather chances in the required location of Sindh. Due to long dry spells dryness was found which has a serious impact on water availability, socio economic factors, viral diseases and environmental problems for entire place. It is required to create the means for the storage of rainfall that can be utilized on the hard time to meet the shortage of water.

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