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The Inter Quartile Range Row Method: An Improved Approach for Finding Basic Feasible Solutions in Transportation Problems

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ABSTRACT

In transportation problems, determining a basic feasible solution (BFS) is critical to start solving optimization problems effectively. This paper proposes a novel method, Inter Quartile Range Row Method (IQRRM), to find the BFS of transportation problems. The method is evaluated by solving hundreds of transportation problems and is compared with traditional methods such as North West Corner Method (NWCM), Least Cost Method (LCM), Vogel's Approximation Method (VAM) and Heuristic Method-2 (HM-2). Results indicate that IQRRM consistently provides better solutions in terms of both computational efficiency and solution quality.

Keywords: Operations Research, Transportation Problems, Basic Feasible Solution, Inter Quartile Range Row method.

INTRODUCTION

Transportation problems are an important class of linear programming problems (LPP) in operations research (O.R). These problems involve finding the most cost-effective way to transport goods from several suppliers to several consumers, subject to supply and demand constraints. The objective is to minimize the transportation cost while satisfying all supply and demand requirements.

To solve a transportation problem (TP), one typically starts with an initial basic feasible solution (IBFS). The quality of the initial BFS is important because it affects the number of iterations needed to reach an optimal solution (OS). Several methods exist for finding BFS, including

1. **North West Corner Method (NWCM)**: A simple, straightforward method that allocates shipments starting from the top-left corner of the transportation matrix (Dantzig., 1951).
2. **Least Cost Method (LCM)**: Allocates shipments to cells with the least transportation cost (Alfred, 1990).
3. **Vogel's Approximation Method (VAM)**: Uses penalty costs to allocate shipments in a way that minimizes the transportation cost (Reinfeld and Vogel 1958).
4. **Heuristics Method-2 (HM-2)**: Uses penalty costs to allocate shipments in a way that minimizes the transportation cost (Kirca, O., & Satir, A., 1990).

Although these methods are widely used, they do not always provide the best initial BFS, leading to inefficient optimization processes. This paper introduces a new approach, the **Inter Quartile Range Row Method (IQRRM)** that enhances the BFS by considering statistical measures, providing better results in comparison to the traditional methods.

METHODOLOGY

INTER QUARTILE RANGE ROW METHOD (IQRRM)

The Inter Quartile Range Row Method (IQRRM) is an innovative technique for finding the BFS by leveraging the interquartile range (IQR) (Guillaume et al., 2024) of each row in the transportation matrix. The IQR is a measure of statistical dispersion, calculated as the difference between the third quartile (Q3) and the first quartile (Q1) (Bhadane, & Manjarekar 2020) of the data in a row. By using IQR, the method balances the distribution of transportation quantities and costs in a more systematic and reliable way than traditional methods.

ALGORITHM (STEPS OF THE IQRRM)

Step1. Balance the Transportation Problem, if needed.
Step2. Find the Inter Quartile Range Row wise:
IQRR = UQ – LQ, where UQ = 3/4 (n+1)th, LQ = 1/4 (n+1)th and n = no. of columns.
Step3. Write the IQRR in front of each row, Select max IQRR & allocate unit of the product to min cost of that row.

a. If tie occurs in maximum IQRR then select minimum unit transportation cost from rows.
b. If tie occurs in unit transportation cost then select minimum supply or demand of that cell.
c. If tie occurs in supply & demand, then select maximum sum of that row or column.

Step4. Eliminate satisfied row or column.
Step5. Repeat process till a row or a column is excavated. Allocation is made to calculate the least cost of transportation.

NUMERICAL EXAMPLES

A. BALANCED TRANSPORTATION LINEAR PROGRAMMING PROBLEM 1:

Find BFS of Transportation Problem using Inter Quartile Range Row Method (IQRRM)?
Table 1 is taken from (Chungath , 2011), (Hosseini, 2017), (Jamali, et al., 2019), (Jamali, et al., 2020).

Table.A1. Example data

	D_1	D_2	D_3	D_4	Supply
S_1	3	5	7	6	50
S_2	2	5	8	2	75
S_3	3	6	9	2	25
Demand	20	20	50	60	

Solution: Solve by Proposed Inter Quartile Range Row Method (IQRRM)
Start transportation problem Table 2 is balanced:

Table A2. Balanced

	D_1	D_2	D_3	D_4	Supply
S_1	3	5	7	6	50
S_2	2	5	8	2	75
S_3	3	6	9	2	25
Demand	20	20	50	60	150

Quantity of supply constraints-3 & Quantity of demand constraints-4

Iteration 1:
XR_1 = 3 5 6 7
n = 4
LQ = ¼(4+1)th= ¼(5) = 5/4 = (1.25)th = (Average of 1st and 2nd) = (3+5)/2 = 8/2 = 4
UQ = ¾ (n+1)th = ¾ (4+1) = 3*5/4 = 15/4 = (3.75)th = (Average of 3rd and 4th) = (6+7)/2 = 13/2 = 6.5
$IQRR_1$ = UQ – LQ = 6.5 – 4 = 2.5

XR_2 = 2 2 5 8
n = 4
LQ = ¼(4+1)th= ¼(5) = 5/4 = (1.25)th = (Average of 1st and 2nd) = (2+2)/2 = 4/2 = 2
UQ = ¾ (n+1)th = Hui¾ (4+1) = 3*5/4 = 15/4 = (3.75)th = (Average of 3rd and 4th) = (5+8)/2 = 13/2 = 6.5
$IQRR_2$ = UQ – LQ = 6.5 – 2 = 4.5

XR_3 = 2 3 6 9
n = 4
LQ = ¼(4+1)th= ¼(5) = 5/4 = (1.25)th = (Average of 1st and 2nd) = (2+3)/2 = 5/2 = 2.5
UQ = ¾ (n+1)th = ¾ (4+1) = 3*5/4 = 15/4 = (3.75)th = (Average of 3rd and 4th) = (6+9)/2 = 15/2 = 7.5
$IQRR_3$ = UQ – LQ = 7.5 – 2.5 = 5

Table A3. After applying IQR row

	D_1	D_2	D_3	D_4	Supply	IQR Row
S_1	3	5	7	6	50	2.5
S_2	2	5	8	2	75	4.5
S_3	3	6	9	2	25	5
Demand	20	20	50	60	150	

The maximum IQR Row = 5, occurs in Row S_3. The minimum c_{ij} in this row is c_{34} =2 . The maximum allocation in this cell is 25. It satisfy supply of S_3 and adjust the demand of D_4 from 60 to 25.
Now delete S_3 row:

Table A4. By deleting the S3 row

	D_1	D_2	D_3	D_4	Supply
S_1	3	5	7	6	50
S_2	2	5	8	2	75
Demand	20	20	50	35	125

Iteration 2:
XR_1 = 3 5 6 7
n = 4
LQ = ¼(4+1)th= ¼(5) = 5/4 = (1.25)th = (Average of 1st and 2nd) = (3+5)/2 = 8/2 = 4
UQ = ¾ (n+1)th = ¾ (4+1) = 3*5/4 = 15/4 = (3.75)th = (Average of 3rd and 4th) = (6+7)/2 = 13/2 = 6.5
$IQRR_1$ = UQ – LQ = 6.5 – 4 = 2.5

XR_2 = 2 2 5 8
n = 4
LQ = ¼(4+1)th= ¼(5) = 5/4 = (1.25)th = (Average of 1st and 2nd) = (2+2)/2 = 4/2 = 2
UQ = ¾ (n+1)th = ¾ (4+1) = 3*5/4 = 15/4 = (3.75)th = (Average of 3rd and 4th) = (5+8)/2 = 13/2 = 6.5
$IQRR_2$ = UQ – LQ = 6.5 – 2 = 4.5

Table A5. IQR row application

	D_1	D_2	D_3	D_4	Supply	IQR Row
S_1	3	5	7	6	50	2.5
S_2	2	5	8	2	75	4.5
Demand	20	20	50	35	125	

The maximum IQR Row = 4.5, occurs in Row S_2. The minimum c_{ij} in this row is c_{21} = 2. The maximum allocation in this cell is 20. It satisfies demand of D_1 and adjusts the supply of S_2 from 75 to 55.
Delete D_1 column.

Table A6. New formation

	D_2	D_3	D_4	Supply
S_1	5	7	6	50
S_2	5	8	2	55
Demand	20	50	35	105

Iteration 3:
XR_1 = 5 6 7
n = 3
LQ = ¼(3+1)th= ¼(4) = 4/4 = 1st = 5
UQ = ¾ (n+1)th = ¾ (3+1) = 3*4/4 = 3rd = 7
$IQRR_1$ = UQ – LQ = 7 – 5 = 2

XR_2 = 2 5 8
n = 3
LQ = ¼(3+1)th= ¼(4) = 4/4 = 1st = 2
UQ = ¾ (n+1)th = ¾ (3+1) = 3*4/4 = 3rd = 8
$IQRR_2$ = UQ – LQ = 8 – 2 = 6

Table A7.

	D_2	D_3	D_4	Supply	IQR Row
S_1	5	7	6	50	2
S_2	5	8	2	55	6
Demand	20	50	35	105	

The maximum IQR Row = 6, occurs in Row S_2. The minimum c_{ij} in this row is c_{23} = 2. The maximum allocation in this cell is 35. It satisfies demand of D_4 and adjust the supply of S_2 from 55 to 20.
Delete D_4 column.

Table A8. New formation

	D_2	D_3	Supply
S_1	5	7	50
S_2	5	8	20
Demand	20	50	70

Iteration 4:
XR_1 = 5 7
n = 2
LQ = ¼(n+1)th= ¼(2+1) = ¼(3) = 3/4 = (0.75)th = 1st = 5
UQ = ¾ (n+1)th = ¾ (2+1) = 3*3/4 = 9/4 = (2.25)th = (Average of 2nd and 3rd) = 2nd = 7
$IQRR_1$ = UQ – LQ = 7 – 5 = 2

XR_2 = 5 8
n = 2
LQ = ¼(n+1)th= ¼(2+1) = ¼(3) = 3/4 = (0.75)th = 1st = 5
UQ = ¾ (n+1)th = ¾ (2+1) = 3*3/4 = 9/4 = (2.25)th = (Average of 2nd and 3rd) = 2nd = 8
$IQRR_2$ = UQ – LQ = 8 – 5 = 3

Table A9. Applying IQR row

	D_2	D_3	Supply	IQR Row
S_1	5	7	50	2
S_2	5	8	20	3
Demand	20	50	70	

The maximum IQR Row = 3, occurs in Row S_2. The minimum c_{ij} in this row is c_{21} = 5. The maximum allocation in this cell is 20. It satisfy supply of S_2 and adjust the demand of D_2 from 20 to 0. Delete S_2 Row.
Hence the new formatted tables (10 and 11) are;

Table A10. New formatted

	D_2	D_3	Supply
S_1	5	7	50
Demand	0	50	50

Table 11. New formatted

	D_2	D_3	Supply
S_1	5	7	50
Demand	0	50	0

The minimum c_{ij} in this Row are C_{11} = 5 and C_{12} = 7.
The maximum allocations in these cells are 0 and 50.
It satisfies demand of D_2 & D_3 & adjusts supply of S_1.

Table A11. Last allocated table of BFS by IQRRM

	D_1	D_2	D_3	D_4	Supply	IQR Row
S 1	3	~~5(0)~~	~~7(50)~~	6	50-50-0=0	---|---|---|---|---|
S 2	~~2(20)~~	~~5(20)~~	8	~~2(35)~~	75-20=55-35=20→0	---|---|---|4|---|
S 3	3	6	9	~~2(25)~~	25→0	1|---|---|---|---|
Demand	20→0	20-20=0→0	50→0	60-25=35→0	0	
Column	---	---	---	---		
	2	---	---	---		
	---	---	---	3		
	---	---	---	---		
	---	6	5	---		

The minimum total cost of transportation = 2 × 25 + 2 × 20 + 2 × 35 + 5 × 20 + 5 × 0 + 7 × 35 = 50 + 40 + 70 + 100 + 0 + 350 = 610. At this stage, the no. of assigned cubicles is 6 i.e. = m + n - 1 = 3 + 4 - 1 = 6. Aforesaid BFS is Non-Degeneracy.
The Proposed Method (IQRRM) of Transportation Problem (TP) solved in 4 iterations, which is Non-Degeneracy BFS, instead of existing standard methods VAM of TP solved in 6 iterations but it is Non-Degeneracy BFS and HM2 of TP also solved in 6 iterations but it is Degeneracy BFS. So, the Proposed new technique is faster than VAM & HM2, where both standard & proposed BFS (HM2 & IQRRM=610) but one standard BFS (VAM=650). Although, all standard & proposed BFS & standard Optimal Method (HM2, IQRRM & MODI=610).

Table A12. Last allocated table of BFS by HM2

	D_1	D_2	D_3	D_4	Sup	Row Penalty
S_1	3	5	7(**50**)	6	50	4 | 4 | 4 | 2 | 0 |
S_2	2(**20**)	5(**20**)	8	2(**35**)	75	6 | 6 | 6 | 3 | -- |
S_3	3	6	9	2(**25**)	25	7 | -- | -- | -- | -- |
Demand	20	20	50	60		
Column Penalty	1 1 1 -- --	1 0 0 0 --	2 1 1 1 0	4 4 -- -- --		

The least total cost of transportation = 7 × 50 + 2 × 20 + 5 × 20 + 2 × 35 + 2 × 25 = 610. At this stage, the no. of assigned cubicles is 5 which is 1 less than m + n – 1 = 3 + 4 – 1 = 6. Aforesaid BFS is Degeneracy.

Table A13. Last Allocated Table of BFS by VAM

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	3(**20**)	5	7(**30**)	6	50	2 | 1 | 1 | 2 | 7 | 7 |
S_2	2	5(**20**)	8(**20**)	2(**35**)	75	0 | 3 | 3 | 3 | 8 | -- |
S_3	3	6	9	2(**25**)	25	1 | 4 | -- | -- | -- | -- |
Demand	20	20	50	60		
Column Penalty	1 -- -- -- -- --	0 0 0 0 -- --	1 1 1 1 1 7	0 0 4 -- -- --		

The least total cost of transportation = 3 × 20 + 7 × 30 + 5 × 20 + 8 × 20 + 2 × 35 + 2 × 25 = 650. At this stage, the no. of assigned cubicles is 6 i.e. = m + n - 1 = 3 + 4 - 1 = 6. Aforesaid BFS is Non-Degeneracy.

Table A14. Last Allocated table of BFS by LCM					
	D_1	D_2	D_3	D_4	Supply
S_1	3 **(5)**	5 **(20)**	7 **(25)**	6	50
S_2	2 **(15)**	5	8	2 **(60)**	75
S_3	3	6	9 **(25)**	2	25
Demand	20	20	50	60	

The least total cost of transportation = 3 × 5 + 5 × 20 + 7 × 25 + 2 × 15 + 2 × 60 + 9 × 25 = 665. At this stage, the no. of assigned cubicles is 6 i.e. = m + n - 1 = 3 + 4 - 1 = 6. Aforesaid BFS is Non-Degeneracy.

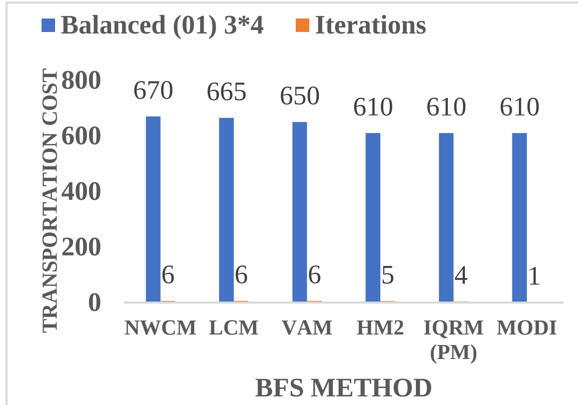
Table A15. Final Allocation Table of BFS by NWCM					
	D_1	D_2	D_3	D_4	Supply
S_1	3 **(20)**	5 **(20)**	7 **(10)**	6	50
S_2	2	5	8 **(40)**	2 **(35)**	75
S_3	3	6	9	2 **(25)**	25
Demand	20	20	50	60	

The least total cost of transportation = 3 × 20 + 5 × 20 + 7 × 10 + 8 × 40 + 2 × 35 + 2 × 25 = 670. At this stage, the no. of assigned cubicles is 6 i.e. = m + n - 1 = 3 + 4 - 1 = 6. Aforesaid BFS is Non-Degeneracy.

BALANCED TRANSPORTATION LINEAR PROGRAMMING PROBLEM 1 RESULTS:

BALANCED TRANSPORTATION LINEAR PROGRAMMING PROBLEM 2

Table A16. Results of analysis for balanced transportation linear programming problem 1			
S.No.	**BFS of Transportation Problem (TP)**	**Balanced (01) 3*4**	**Iterations**
1	**NWCM**	**670**	**6**
2	**LCM**	**665**	**6**
3	**VAM**	**650**	**6**
4	**HM2**	**610**	**5**
5	**PM**	**610**	**4**
6	**MODI**	**610**	**1**



Find BFS of Transportation Problem using Inter Quartile Range Row Method (IQRRM)? A company manufactures motor tyres and it has four factories F1, F2, F3, F4 whose weekly production capacities are 5, 8, 7 & 14 thousand pieces of tyres respectively. The company supplies tyres to its three showrooms located at D1, D2 & D3 whose weekly demand are 7, 9 & 18 thousand pieces respectively. The transportation cost per thousand pieces of Tyree is given below Table B1 is taken from (Ahmed et al., 2014; Ahmed et al., 2016 Chungath L., 2011; Kantharaj, 2018; Mamidi & Murthy, 2014; Mollah 2017; [14] Quddoos, et al., 2016; Shenofer et al., 2017)?

Table B1. Example data				
	D_1	D_2	D_3	Capacities
F_1	2	7	4	5
F_2	3	3	1	8
F_3	5	4	7	7
F_4	1	6	2	14
Requirement	7	9	18	

Quantity of Capacities constraints-4 & Quantity of Requirement constraints-3.

Solution: Solve by Proposed Inter Quartile Range Row Method (IQRRM)

Start transportation problem *Table B2* is balanced Quantity of Capacities constraints-4 & Quantity of Requirement constraints-3

Table B2. Transportation problem				
	D_1	D_2	D_3	Capacities /Supply
F_1	2	7	4	5
F_2	3	3	1	8
F_3	5	4	7	7
F_4	1	6	2	14
Requirement /Demand	7	9	18	34

<table>
<caption>Table B3. Last Allocated of BFS by IQRRM</caption>
<tr><th></th><th>D_1</th><th>D_2</th><th>D_3</th><th>Capacities</th><th>IQR Row</th></tr>
<tr><td>F_1</td><td>~~2(5)~~</td><td>7</td><td>4</td><td>5➔0</td><td>1|---|---|---|---|</td></tr>
<tr><td>F_2</td><td>3</td><td>~~3(2)~~</td><td>~~1(6)~~</td><td>8-2-6=0</td><td>---|---|---|---|---|</td></tr>
<tr><td>F_3</td><td>5</td><td>~~4(7)~~</td><td>7</td><td>7➔0</td><td>---|---|---|4|---|</td></tr>
<tr><td>F_4</td><td>~~1(2)~~</td><td>6</td><td>~~2(12)~~</td><td>14-2=12➔0</td><td>---|---|3|---|---|</td></tr>
<tr><td>Requirement</td><td>7-5=2➔0</td><td>9-7=2➔0</td><td>18-12=6➔0</td><td>0</td><td></td></tr>
<tr><td>Column</td><td>---
2

---</td><td>---

5</td><td>---

6</td><td></td><td></td></tr>
</table>

Last Allocated *Table B3* of BFS by IQRRM:
The minimum total cost of transportation = 2 × 5 + 3 × 2 + 1 × 6 + 4 × 7 + 1 × 2 + 2 × 12 = 10 + 6 + 6 + 28 + 2 + 24 = 76. At this stage, the no. of assigned cubicles is 8, i.e. = m + n – 1 = 4 + 5 – 1 = 8. Aforesaid BFS is Non-Degeneracy.

The Proposed Method (IQRRM) of Transportation Problem (TP) solved in 6 iterations, which is Non-Degeneracy BFS, instead of existing classical methods VAM of TP solved in 6 iterations but it is Non-Degenerate BFS and HM2 of TP also solved in 6 iterations but it is also Non-Degenerate BFS. Therefore, the Proposed new technique is faster than VAM & HM2, where proposed BFS (IQRRM=76) but classical BFS (NWCM=102, VAM=80=ND & LCM=HM2=83). Although, both proposed BFS & classical Optimal Methods (IQRRM, MODI& SSM=76). Hence it is improved to the BFS (VAM=80) & it is Optimal Solution.

<table>
<caption>Table B4. Last allocation of BFS by HM2</caption>
<tr><th></th><th>D_1</th><th>D_2</th><th>D_3</th><th>Capacities</th><th>HM Row</th></tr>
<tr><td>F_1</td><td>2(1)</td><td>7</td><td>4(4)</td><td>5</td><td>---|---|---|---|---|</td></tr>
<tr><td>F_2</td><td>3</td><td>3(8)</td><td>1</td><td>8</td><td>1|---|---|---|---|</td></tr>
<tr><td>F_3</td><td>5(6)</td><td>4(1)</td><td>7</td><td>7</td><td>---|---|---|4|---|</td></tr>
<tr><td>F_4</td><td>1</td><td>6</td><td>2(14)</td><td>14</td><td>---|---|3|---|---|</td></tr>
<tr><td>Requirement</td><td>7</td><td>9</td><td>18</td><td></td><td></td></tr>
<tr><td>HM Column</td><td>---

5</td><td>---
2

---</td><td>---

6</td><td></td><td></td></tr>
</table>

The least total cost of transportation = 2 × 1 + 4 × 4 + 3 × 8 + 5 × 6 + 4 × 1 + 2 × 14 = 2 + 16 + 24 + 30 + 4 + 28 = 104. At this stage, the no. of assigned cubicles is 6, which is equal to m + n – 1 = 4 + 3 – 1 = 6. Aforesaid IBFS is Non-Degeneracy.

Table B5. Last Allocated of IBFS by NWCM

	D_1	D_2	D_3	Supply
F_1	2	7 (2)	4 (3)	5
F_2	3	3	1 (8)	8
F_3	5	4 (7)	7	7
F_4	1 (7)	6	2 (7)	14
Demand	7	9	18	

The least total cost of transportation = 7 × 2 + 4 × 3 + 1 × 8 + 4 × 7 + 1 × 7 + 2 × 7 = 83. At this stage, the quantity of assigned cubicles is 6, which is equal to m + n – 1 = 4 + 3 – 1 = 6. Aforesaid IBFS is Non-Degeneracy.
The least total cost of transportation = 2 × 3 + 7 × 2 + 1 × 8 + 4 × 7 + 1 × 4 + 2 × 10 = 80. At this stage, the quantity of assigned cubicles is equal to 6, which is equal to m + n – 1 = 4 + 3 – 1 = 6. Aforesaid IBFS is Non-Degeneracy (Table B6).

<table>
<caption>Table B6. Last Allocated of IBFS by VAM</caption>
<tr><th></th><th>D_1</th><th>D_2</th><th>D_3</th><th>Supply</th><th>Row Penalty</th></tr>
<tr><td>F_1</td><td>2</td><td>7(2)</td><td>4(3)</td><td>5</td><td>5 | 5 | 3 | 3 | 3 | 0 |</td></tr>
<tr><td>F_2</td><td>3</td><td>3</td><td>1(8)</td><td>8</td><td>2 | -- | -- | -- | -- | -- |</td></tr>
<tr><td>F_3</td><td>5</td><td>4(7)</td><td>7</td><td>7</td><td>3 | 3 | 3 | 3 | -- | -- |</td></tr>
<tr><td>F_4</td><td>1(7)</td><td>6</td><td>2(7)</td><td>14</td><td>5 | 5 | 4 | -- | -- | -- |</td></tr>
<tr><td>Demand</td><td>7</td><td>9</td><td>18</td><td></td><td></td></tr>
<tr><td>Column Penalty</td><td>4
4
--
--
--
--</td><td>4
3
3
3
0
0</td><td>6
5
5
3
0
--</td><td></td><td></td></tr>
</table>

The least total cost of transportation = 2 × 3 + 7 × 2 + 1 × 8 + 4 × 7 + 1 × 4 + 2 × 10 = 80. At this stage, the quantity of assigned cubicles is equal to 6, which is equal to m + n – 1 = 4 + 3 – 1 = 6. Below IBFS is Non-Degeneracy (Table B7).

Table B7. Last Allocated of IBFS by LCM

	D_1	D_2	D_3	Supply	Row Penalty
F1	2(3)	7(2)	4	5	2 | 2 | 5 | 5 | 7 | -- |
F2	3	3	1(8)	8	2 | -- | -- | -- | -- | -- |
F3	5	4(7)	7	7	1 | 1 | 1 | 1 | 4 | 4 |
F4	1(4)	6	2(10)	14	1 | 1 | 5 | -- | -- | -- |
Demand	7	9	18		
Column Penalty	1 1 1 3 -- --	1 2 2 3 3 4	1 2 -- -- -- --		

BALANCED TRANSPORTATION LINEAR PROGRAMMING PROBLEM 2 RESULTS.

Table B8. Results of analysis

S.#	IBFS of Transportation Problem (TP)	Balanced (02) 4*5	Iterations
1	**NWCM**	**102**	**6**
2	LCM	**83**	**6**
3	VAM	**80**	**6**
4	HM2	**83**	6
5	IQRM (PM)	**76**	**4**
6	MODI	**76**	**1**

C. UNBALANCED TRANSPORTATION LINEAR PROGRAMMING PROBLEM 3

Find BFS of Transportation Problem using Inter Quartile Range Row Method (IQRRM)?

Table C1. Data

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	20	25	27	20	15	40
S_2	18	27	22	24	20	70
S_3	19	17	20	18	19	90
Demand	30	40	60	40	60	

Solution: Solve by Proposed Inter Quartile Range Row Method (IQRRM)
Start transportation problem Table C1 but problem is unbalanced.

Table C2.

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	20	25	27	20	15	40
S_2	18	27	22	24	20	70
S_3	19	17	20	18	19	90
Demand	30	40	60	40	60	230/200

Sum of Demand = 230 is not equal to the sum of Supply = 200. So, now adding of dummy supply using unit cost as zero & portion 30.
Quantity of supply constraints-3 & Quantity of demand constraints-5.
The Modified Table C2 of problem is balanced:

Table C3. Modified table after balancing

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	20	25	27	20	15	40
S_2	18	27	22	24	20	70
S_3	19	17	20	18	19	90
S dummy	0	0	0	0	0	30
Demand	30	40	60	40	60	230

Quantity of supply constraints-3 & Quantity of demand constraints-5.
According to Table C4 below, the minimum total cost of transportation = 15 × 40 + 18 × 30 + 22 × 20 + 20 × 20 + 17 × 40 + 20 × 10 + 18 × 40 + 0 × 30 = 600 + 540 + 440 + 400 + 680 + 200 + 720 + 0 = 3580. At this stage, the no. of assigned cubicles is 8, i.e. = m + n – 1 = 4 + 5 – 1 = 8. Aforesaid BFS is Non-Degeneracy.
The Proposed Method (IQRRM) of Transportation Problem (TP) solved in 6 iterations, which is Non-Degeneracy BFS, instead of existing standard methods VAM of TP solved in 8 iterations but it is Non-Degeneracy BFS and HM2 of TP also solved in 8 iterations but it is also Non-Degeneracy BFS. So, the Proposed new technique is faster than VAM & HM2, where both standard & proposed BFS (VAM & IQRRM=3580) but standards BFS (HM-2=3670, LCM=3710 but Degeneracy and NWCM=4290). Although, both standards (BFS and OS) & proposed BFS (HM2, MODI & IQRRM=3580).

Table C4. The last allocated table of BFS by IQRRM							
	D_1	D_2	D_3	D_4	D_5	Supply	IQR Row
S_1	20	25	27	20	~~15(40)~~	40=0	1|---|---|---|---|---|---|---|
S_2	~~18(30)~~	27	~~22(20)~~	24	~~20(20)~~	70-30=40-20=20=0	---|---|---|4|---|---|---|
S_3	19	~~17(40)~~	~~20(10)~~	~~18(40)~~	19	90-40=50-40=10=0	---|---|---|---|---|---|7|
S_{dummy}	0	0	~~0(30)~~	0	0	30=0	---|---|---|---|---|---|8|
Demand	30=0	40=0	60-20=40-10-30=0	40=0	60-40=20=0	0	
Column	--- 2 --- --- --- --- ---	--- --- --- --- 5 --- ---	--- --- --- --- --- --- ---	--- ---- --- --- --- 6 ---	--- --- 3 --- --- --- ---		

The least total cost of transportation = 15 × 40 + 18 × 30 + 22 × 20 + 20 × 20 + 17 × 10 + 20 × 40 + 18 × 40 + 0 × 30 = 3670. At this stage, the no. of assigned cubicles is 8, which is equal to m + n - 1 = 4 + 5 - 1 = 8. Aforesaid BFS is Non-Degeneracy (Table C5).

The least total cost of transportation = 15 × 40 + 18 × 30 + 22 × 20 + 20 × 20 + 17 × 40 + 20 × 10 + 18 × 40 + 0 × 30 = 3580. At this stage, the no. of assigned cubicles is 8 which is equal to m + n – 1 = 4 + 5 – 1 = 8. Aforesaid BFS is None-Degeneracy (Table C6).

Table C5. Last Allocated table of BFS by HM2							
	D_1	D_2	D_3	D_4	D_5	Supply	Row Penalty
S_1	20	25	27	20	15**(40)**	40	12 | 12 | -- | -- | -- | -- | -- | -- |
S_2	18**(30)**	27	22**(20)**	24	20**(20)**	70	9 | 9 | 9 | 6 | 4 | 2 | 2 | 0 |
S_3	19	17**(10)**	20**(40)**	18**(40)**	19	90	3 | 3 | 3 | 2 | 1 | 1 | -- | -- |
S_{dummy}	0	0**(30)**	0	0	0	30	0 | -- | -- | -- | -- | -- | -- | -- |
Demand	30	40	60	40	60		
Column Penalty	20 2 1 1 1 -- -- --	27 10 10 -- -- -- -- --	27 7 2 2 2 2 0 0	24 6 6 6 -- -- -- --	20 5 1 1 1 1 0 --		

Table C6. Last Allocated table of BFS by VAM							
	D_1	D_2	D_3	D_4	D_5	Supply	Row Penalty
S_1	20	25	27	20	15**(40)**	40	5 | 5 | 5 | -- | -- | -- | -- | -- |
S_2	18**(30)**	27	22**(20)**	24	20**(20)**	70	2 | 2 | 2 | 2 | 2 | 2 | 22 | -- |
S_3	19	17**(40)**	20**(10)**	18**(40)**	19	90	1 | 1 | 1 | 1 | 0 | 1 |20 | 20 |
S_{dummy}	0	0	0**(30)**	0	0	30	0 | -- | -- | -- | -- | -- | -- | -- |
Demand	30	40	60	40	60		
Column Penalty	18 1 1 1 1 -- -- --	17 8 -- -- -- -- -- --	20 2 2 2 2 2 2 20	18 2 2 6 -- -- -- --	15 4 4 1 1 1 -- --		

The least total cost of transportation = 15 × 40 + 22 × 60 + 20 × 10 + 17 × 40 + 18 × 40 + 19 × 10 + 0 × 30 = 3710. At this stage, the quantity of assigned cubicles is equal to 7, which is two less than to m + n – 1 = 4 + 5 – 1 = 8. Aforesaid BFS is Degeneracy.

To determine degeneracy, we utilize a simulated amount (q). The amount q is allotted to that null cubicle, which has the smallest transportation cost. The amount q is allotted on the way to S2D1, which has the smallest transportation cost = 18.

Table C7. Last Allocated table of BFS by LCM						
	D_1	D_2	D_3	D_4	D_5	Supply
S_1	20	25	27	20	15 **(40)**	40
S_2	18	27	22 **(60)**	24	20 **(10)**	70
S_3	19	17 **(40)**	20	18 **(40)**	19 **(10)**	90
S_{dummy}	0 **(30)**	0	0	0	0	30
Demand	30	40	60	40	60	

The least total cost of transportation = 20 × 30 + 25 × 10 + 29 × 30 + 22 × 40 + 20 × 20 + 18 × 40 + 19 × 30 + 0 × 30 = 4290. At this stage, the quantity of assigned cubicles is 8, which is equal to m + n – 1 = 4 + 5 – 1 = 8. Aforesaid BFS is Non-Degeneracy.

UNBALANCED TRANSPORTATION LINEAR PROGRAMMING PROBLEM 3: RESULTS.

S.#	BFS of Transportation Problem (TP)	Unbalanced B in R (03) 4*5	Iterations
1	**NWCM**	4290	**8**
2	LCM	3710	**8**
3	VAM	3580	**8**
4	HM2	3670	**8**
5	IQRM (PM)	**3580**	**6**
6	MODI	**3580**	**1**

COMPARATIVE ANALYSIS

To evaluate the effectiveness of the **Inter Quartile Range Row Method (IQRRM)**, it was tested on a variety of transportation problems of different sizes and complexities. The performance of IQRRM was compared with four widely used methods:

North West Corner Method (NWCM): A simple but sometimes inefficient method that often results in high initial transportation costs.

Least Cost Method (LCM): Allocates shipments based on the least cost available in each cell, which tends to provide a more cost-efficient solution than NWCM but is still limited in its ability to find the optimal BFS.

Vogel's Approximation Method (VAM): A more sophisticated method that minimizes penalties by considering the difference between the two least costs in each row and column. It often yields better initial BFS solutions compared to NWCM and LCM.

Heuristic Method-2 (HM-2): A more sophisticated method that maximizes penalties by considering the difference between the two least costs in each row and column and with minimize transportation cost of which allocated selected penalty and satisfy row or column. It often yields better initial BFS solutions compared to VAM, NWCM and LCM.

EXPERIMENTAL SETUP

Number of Problems: The methods were tested on hundreds of transportation problems with varying numbers of supply and demand points, ranging from small (4x4) to large (10x10) matrices.

Table C8. Non-degeneracy modified table of BFS b LCM						
	D_1	D_2	D_3	D_4	D_5	Supply
S_1	20 1	25 1	27 1	20 1	15 **(40)** 2	40
S_2	18 **(q)** 1	27 1	22 **(60)** 4	24 1	20 **(10)** 3	70
S_3	19 1	17 **(40)** 4	20 1	18 **(40)** 4	19 **(10)** 2	90
S_{dummy}	0 **(30)**	0 1	0 1	0 1	0 1	30
Demand	30	40	60	40	60	

Table C9. Last allocated Table of BFS by NWCM						
	D_1	D_2	D_3	D_4	D_5	Supply
S_1	20 **(30)**	25 **(10)**	27	20	15	40
S_2	18	29 **(30)**	22 **(40)**	24	20	70
S_3	19	17	20 **(20)**	18 **(40)**	19 **(30)**	90
S_{dummy}	0	0	0	0	0 **(30)**	30
Demand	30	40	60	40	60	

Performance Metrics: The primary performance metric used for comparison was the total transportation cost of the initial BFS. Additionally, computational efficiency, measured by the time taken to compute the BFS, was also analyzed.

RESULTS

The results of the experiment showed the following:

Total Transportation Cost: The IQRRM outperformed NWCM, LCM, VAM and HM-2 in terms of reducing the total transportation cost in the initial BFS. On average, IQRRM resulted in 10% lower transportation costs than HM-2 and VAM and 20% lower than LCM.

Computational Efficiency: In terms of computational time, IQRRM performed similarly to VAM, with only a slight increase in time due to the additional IQR calculation. However, the efficiency gain in terms of cost reduction justifies the marginal increase in time.

Consistency: IQRRM provided more consistent and reliable results across a broad range of problem types compared to the other methods, which sometimes yielded suboptimal solutions depending on the matrix structure.

DISCUSSION

The results of this study highlight several advantages of the **Inter Quartile Range Row Method (IQRRM)** over traditional methods such as **NWCM**, **LCM**, **VAM** and **HM-2**:

Better Cost Efficiency: IQRRM consistently provides better BFS solutions in terms of transportation cost. This can lead to faster convergence in optimization methods like the **MODI method** or **Stepping Stone**

Method, which are used to refine the BFS to an optimal solution.

Adaptability: The statistical basis of IQRRM makes it adaptable to different problem sizes and structures, providing a more robust approach to finding the BFS in diverse scenarios.

Simplicity: While incorporating statistical analysis, the IQRRM remains relatively simple to implement compared to more complex methods like VAM, which requires penalty calculations.

However, one limitation of IQRRM is its slight increase in computational time compared to methods like **NWCM** and **LCM**, which might be significant for extremely large problems. Future research could explore further optimization of the IQRRM to address this issue.

Table 10. Comparison Table of Proposed & Various Methods							
S. #	ORDER OF PROBLEM	NWCM	LCM	VAM	HM2	MODI	PM
1.	BTP 3*4	670	665	650	610	610	610
2	BTP 4*5	102	83	80	83	76	76
3	UTP 4*5 B in R	4290	3710	3580	3670	3580	3580

CONCLUSION

The **Inter Quartile Range Row Method (IQRRM)** offers a promising new approach for finding basic feasible solutions to transportation problems. It outperforms traditional methods like **North West Corner Method (NWCM)**, **Least Cost Method (LCM)**, **Vogel's Approximation Method (VAM)** and **Heuristic Method-2 (HM-2)** in terms of minimizing transportation costs and providing a more consistent solution. This method offers an important step forward in the field of transportation problem solving, particularly for large-scale and complex problems where traditional methods may fall short.

CONFLICT OF INTEREST

The authors declare no conflict of interest for this research.

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