Sindh Univ. Res. Jour. (Sci. Ser.) Vol. 40 (2) 85-88 (2008)



Sindh Univ. Res. Jour.

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SEARCHING OF OPTIMAL VERTICES IN A HAPHAZARD ROUTES

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(Received 12th March 2008 and Revised 09th May 2008)

Abstract

In this paper we are going to develop a new and efficient algorithm for finding a shortest cyclic tour of the network with the repetition of each vertex at least once. Problem under consideration is a **traveling salesman problem**, for repeating vertex. Penalizing the distance and choosing the shortest tour has been achieved from the weighted matrix of the graph.

Keywords: Traveling salesman problem, Optimal salesman problem

1. Introduction

Finding the shortest tour within the city or area is a very useful requirement. The optimality is defined in terms of the cost it takes to complete the traversal. This cost depends on the number of times of each edge traversed through out the tour.

In this paper we proposed a new algorithm to find the optimal tour in a directed network, called *Asymmetric optimal sales man problem.*

Our algorithm is based on traveling sales man problem for repeated vertex, first we shall find all possible tours that each starts and ends at fixed point, then we combine all these tours, in the end we apply the removal and insertion to find the optimal tour. Solving TSPs is an important part of application in many areas including vehicle routing, computer wiring, machine sequencing and scheduling, frequency assignment in communication network and structuring of matrices (Lenstra and Kan, 1975; Punnen, 2002). Given a graph network, finding the tour of minimum cost (e.g. time or distance) that visits each node of the graph at least once is called *optimal sales man problem.*

The literature on the graph theory includes many variants of the problem of graph traversal with respect to different constraints. One of the classic problems in graph theory is the problem of *Chinese postman tour (CPT)* in which a post man needs to travel along every road, with the least possible cost to deliver mails. Another problem is the *Traveling sales man problem* (TSP) Given a collection for cities and the cost of travel between each city pair, TSP is to find the cheapest way of visiting of all the cities and retuning to the starting point (i.e, finding a Hamiltonian cycle with the least weight). If the distance between the cities is not symmetric (i.e d (city 1, city 2)≠d (city 2, city 1), the problem is referred to as the *Asymmetric Traveling Sales Man Problem (ATSP).* Another approach (Bellmore and Malone) extant the solution of TSP is *Branch and Bound* in which first find the Hamiltonian cycle of the given network, many variations has been made in branch and bound method to solve TSP. This method is also used for optimal salesman problem with some difference we use shortest distance matrix (SD) instead of weighted matrix. In our algorithm we are also using this SD matrix. Another method (Little *et al* 1963) called weight matrix reduction is also could be used to solve TSP and OSP. An algorithm is also used to solve TSP for the curvature constrained.

The remainder of the paper is organized as follows. Section 2 formally define the OPT problem. In section 3, we proposed the algorithm. Section 4 includes the experimental problem and section 5 discusses the conclusion.

2. Definitions

The simplest model used to analyze facility location problem is a graph. A graph G consists of a finite nonempty set $V = V(G)$ of

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vertices together with the set $E = E(G)$ of line segments (called edges) joining pairs of distinct vertices of V. If $e_k \in E$, then v_i and v_j are adjacent vertices, otherwise, they are non-adjacent. The *degree* of a vertex is its number of adjacent vertices. A *path* in G is an alternating sequence $v_0 e_1 v_1 \ldots v_{k-1} e_k v_k$ of distinct vertices and edges such that $e_k = v_{k-1} v_k$ is an edge of G. Since each edge is uniquely determined by its end vertices, it is customary to write $v_0 v_1 \ldots v_{k-1} v_k$ for the path, just described. A path $v_0 v_1 \ldots v_{k-1} v_k v_0$ where v_0 and v_k are joined by an edge, is called a *cycle*. A graph G is connected if there exists a path joining each pair of vertices in G. Thus, if a graph modeling a telecommunication network is connected, we know that it is possible for any two devices in the network to communicate. We shall restrict our attention to connected graph. The matrix representation of a graph, in which element at i^{th} row and j^{th} column represents length of e_{ij} is called the *Weighted matrix* of that graph.

The *length of a path* is the sum of length of edges it contains. The *distance d(i,j)* is the length of the shortest path joining v_i and v_j. The distance between v_i to itself is to be considered as ∞ i.e. $d(i,i) = \infty$. A matrix obtained after row and column reduction of a matrix of a graph is called *reduced weight matrix*, where row reduction is the subtraction of least element from each element of the row and column reduction is the subtraction of the least element from each element of the column.

3. Algorithm

i. Input : weighted matrix : D,
D': Reduced weighted matrix,
T_i Sets of all possible tours.

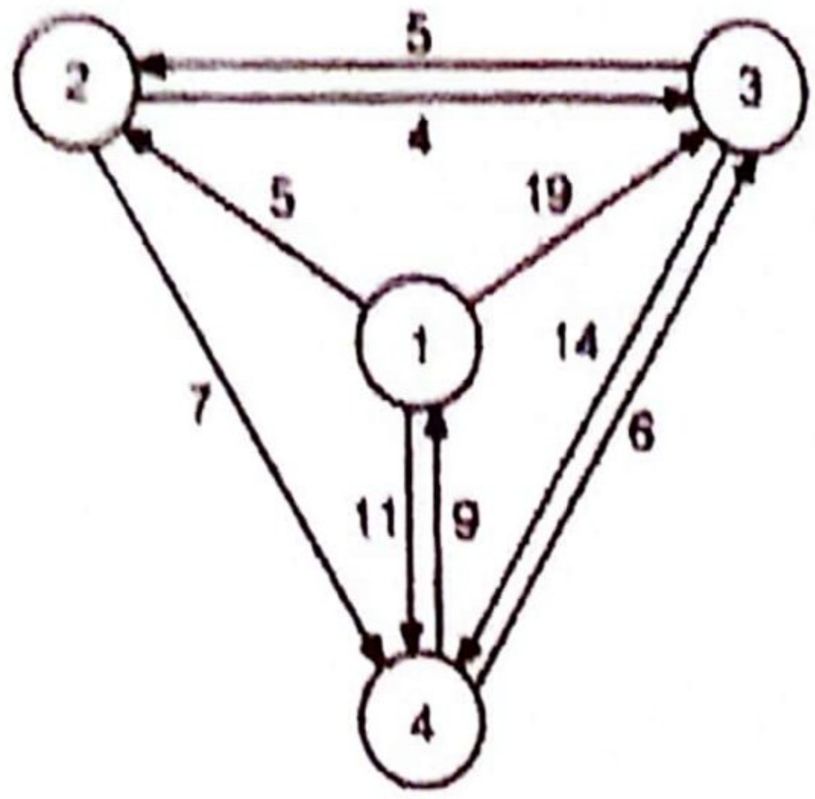
ii. For D' : Row/column penalties.

iii. For $\{T_i\}$: Starting from a $_{ij}$
Selecting $e_{ij=0}$ if e_{ij} already visited then select next minimum e_{ij}.

iv. Union S = $S = \sum T_i$

v. Redundant path removal for each k by removing all paths $v_i v_k v_j$ except that of min $[(e_{ik} + e_{kj}) - e_{ij}]$.

vi. Insertion of path $v_i v_k v_j$ If $[(e_{ik} + e_{kj}) < e_{ij}]$.
Return vector for shortest tour.



4. Application of the constructed model

For the given graph find the weighted matrix of the network with diagonal elements ∞. Find the reduced weighted matrix by applying row and column reduction operation. Starting from the first vertex find all possible tour by selecting zero elements in the reduced weighted matrix, if there is no edge of zero value of available to complete such tour the edge with next most minimum value will be selected instead. Combined all the possible tours. After the combination of all the tour, start with a tour consisting of an arbitrary city and then choose in each step a city k. This city is deleted in to the existing tour between city i and j such that the deletion cost (i.e the decrease in the tour's length)

$$(e_{ik} + e_{kj}) - e_{ij}$$

is maximized. The algorithm stops when all the cities on the tour (just once), after that we insert the city between the two consecutive cities i and j such that if

$$(e_{ik} + e_{kj}) < e_{ij}$$

by using SD matrix. In this way the OSP tour of the given network is

$$1 \rightarrow 2 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 1$$

with the total cost 30, in contrast with the TSP result

$$1 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 1$$

with the total cost 40.

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5. Conclusion

In this paper, we studied the optimal salesman problem and proposed a new algorithm in OSP, by considering the weighted matrix of the graph. First we have calculated all the possible tours, then we have combined all these tours, in the end we have applied the removal & insertion to get the optimal vertices in haphazard routes. Our example showed the superiority of OSP as compare to TSP.

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