Sind Univ. Res. Jour. (Sci. Ser.) Vol: 38(1) 33 -38 (2006)

Evaluation of an Advertising Budget, a Game Theoretical Model

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Received July 2005, Revised March 2006

Abstract

This paper describes a two opponents non–zero sum game model for advising a group of cellular companies to economise their expenses uplift profits and increase market share. Various confines and assumptions form the basis of this model; let each company of the group provide similar facilities and essential of these being rigid demand both these groups must have identical cost and related price structure. This model is likely to lay down a basis for a more comprehensive modeling.

Keywords: game theory, saddle point

Introduction

We were approached by two different groups of cellular companies with the intention to launch their services in the country. One group was using AMPS (Advance Mobile Phone System) technology while the other GSM (Global System for Mobile) technology. The marketing trend and advertising play a key role in the financial decision making. Although a definite conclusion implies that money spending at appropriate time (quarterly, biannual or annual basis) and in right direction is the focus of capturing market share and increase clientele appreciably.

In this model we restrict each company of the group to provide same facilities, and strictly demand that both competitors have equal opportunities, ignoring other factors for smoothness and considering that uplift of profit and clientele of a specific group depends only on the expenses incurred by both the groups.

For allocation of advertising expenditure, Friedman (1958) proposed an economic model and Brams and Davis (1973) proposed a presidential election campaign model as a game theoretic point of view. Shane (1977) developed Friedman's model for two companies competing in a market in which demand is fully rigid and market is initially divided between only two competitors. We consider here a model that underlines the effect of advertising expenses on share (number of customers) of a group in a competitive condition and ask it about the size of advertising budget in order to maximize profits. In the model we assume that we have only two competing groups GSM and AMPS.

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The Model

Let

S_1 : Current share of GSM group in the market

S_2 : Current share of AMPS group in the market

S_1^* : Expected share of GSM group in next quarter

S_2^* : Expected share of AMPS group in next quarter

x : Expenditure for advertisement by group GSM

y : Expenditure for advertisement by group AMPS

n : Total number of current cellular phone users

n^* : Total number of expected cellular phone users in next quarter

In observance Shane (1977) and Khan (2003) the expected share for both groups in the market respectively is

$$S_1^* = S_1 + \frac{(x-y)}{(x+y)} S_1 S_2 \qquad \text{----------(A)}$$

and

$$1-S_1^* = S_2^* = S_2 + \frac{(y-x)}{(y+x)} S_1 S_2 \qquad \text{----------(B)}$$

The number of expected customers for group GSM will be $n^* S_1^*$ and for group AMPS, $n^* S_2^*$ assuming both groups to have identical cost and connection price structures.

For simplicity let $n = n^*$

Let

a_1: The fixed costs exclusive of advertisement,
a_2: The service cost per connection,
a_3: The connection price and
a_4: The proceeds (revenue) per connection

Then the expected profit for GSM group is P_{GSM} given by

$$P_{GSM} = a_3 n (S_1^* - S_1) - a_1 - a_2 n S_1^* - x + n S_1^* a_4 \qquad \text{----------(1)}$$

And for AMPS group

$$P_{AMPS} = a_3 n (S_2^* - S_2) - a_1 - a_2 n S_2^* - y + n S_2^* a_4 \qquad \text{----------(2)}$$

Let

n_1 : Number of current GSM users

n_2 : Number of current AMPS users

n_1^* : Number of expected GSM users in next quarter

n_2^* : Number of expected AMPS users in next quarter

$$S_1 = \frac{n_1}{n}, \quad S_2 = \frac{n_2}{n} \quad \text{and} \quad S_1^* = \frac{n_1^*}{n}, \quad S_2^* = \frac{n_2^*}{n}$$

Then

$$P_{GSM} = a_3 (n_1^* - n_1) - a_1 - a_2 n_1^* - x + a_4 n_1^* \quad \text{----------(3)}$$

$$P_{AMPS} = a_3 (n_2^* - n_2) - a_1 - a_2 n_2^* - y + a_4 n_2^* \quad \text{----------(4)}$$

Let $B = (a_3 - a_2 + a_4)$ then (3) and (4) can be written as

$$P_{GSM} = Bn_1 + B\frac{n_1 n_2}{n}\frac{(x-y)}{(x+y)} - a_3 n_1 - a_1 - x$$

$$P_{AMPS} = Bn_1 + B\frac{n_1 n_2}{n}\frac{(y-x)}{(y+x)} - a_3 n_1 - a_1 - y$$

Let $C = (B - a_3)$ then

$$P_{GSM} = Cn_1 + B\frac{n_1 n_2}{n}\frac{(x-y)}{(x+y)} - a_1 - x \quad \text{---------- (5)}$$

$$P_{AMPS} = Cn_2 + B\frac{n_1 n_2}{n}\frac{(y-x)}{(y+x)} - a_1 - y \quad \text{---------- (6)}$$

To theoretically determine exact saddle point, we will consider it as a two competitor game in which objective of GSM group is to maximize P_{GSM} and objective of AMPS group is to maximize P_{AMPS}. For this purpose we use elementary calculus to seek the optimal strategies.

$$(5) \Rightarrow \quad \frac{\partial P_{GSM}}{\partial x} = \frac{2Bn_1 n_2 y}{n(x+y)^2} - 1 = 0$$

$$\Rightarrow \quad x = \left(\frac{2Bn_1 n_2 y}{n}\right)^{\frac{1}{2}} - y \quad \text{---------- (7)}$$

Equation (7) describes the optimal spending by GSM group for any choice of y. substituting this value of x in (6), we get,

$$P_{AMPS} = Cn_2 - B\frac{n_1 n_2}{n} + \left(\frac{2Bn_1 n_2 y}{n}\right)^{\frac{1}{2}} - a_1 - y \text{ --------- (8)}$$

[Similarly, the optimal spending for AMPS group for any choice of x, we get

$$P_{GSM} = Cn_1 - B\frac{n_1 n_2}{n} + \left(\frac{2Bn_1 n_2 x}{n}\right)^{\frac{1}{2}} - a_1 - x \text{ ---------- (9)}$$

Now we maximize P_{AMPS} (8) by solving

$$\frac{\partial P_{AMPS}}{\partial y} = \frac{(Bn_1 n_2 / n)^{\frac{1}{2}}}{(2y)^{\frac{1}{2}}} - 1 = 0$$

$$\Rightarrow \quad y = \frac{Bn_1 n_2}{2n}$$ Substituting in (7), we get

$$x = \frac{Bn_1 n_2}{2n}$$ as well.

Thus this point $x = \frac{Bn_1 n_2}{2n}$, $y = \frac{Bn_1 n_2}{2n}$ is a true saddle point solution.

Application of model

As we have obtained saddle point for optimal spending of GSM and AMPS group, if a_1= 300, a_2=500, a_3=500, a_4=2000, n=1000, n_1=550 and n_2=450, we have,

$$x = 247500, \quad y = 247500$$

We consider above values for calculating profit,
using (8) & (9) we get,

$$P_{GSM} = 577200 \quad \text{and} \quad P_{AMPS} = 427200$$

We can also observe the above saddle point through the graphs of equations (8) and (9) (See Fig. (a) and (b)).

Conclusion

The present model formulates a basic competitive condition in a market in which demand is fully rigid and the market has only two stake holders. To determine exact saddle point, we have considered it as a two competitors game applying elementary calculus to seek the

optimal strategy. An application of the model is demonstrated through a saddle point implying thereby that any other strategy would result in financial losses. The model can be suitably modified to incorporate more that two competitors.

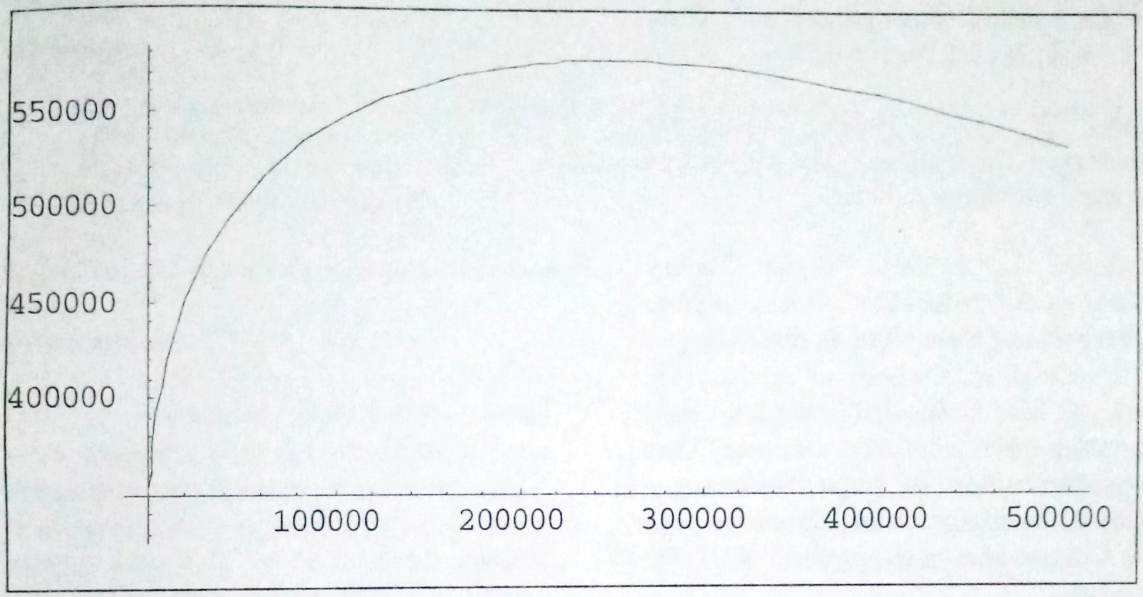


Figure: (a) Profit according to Expenditure for GSM Group

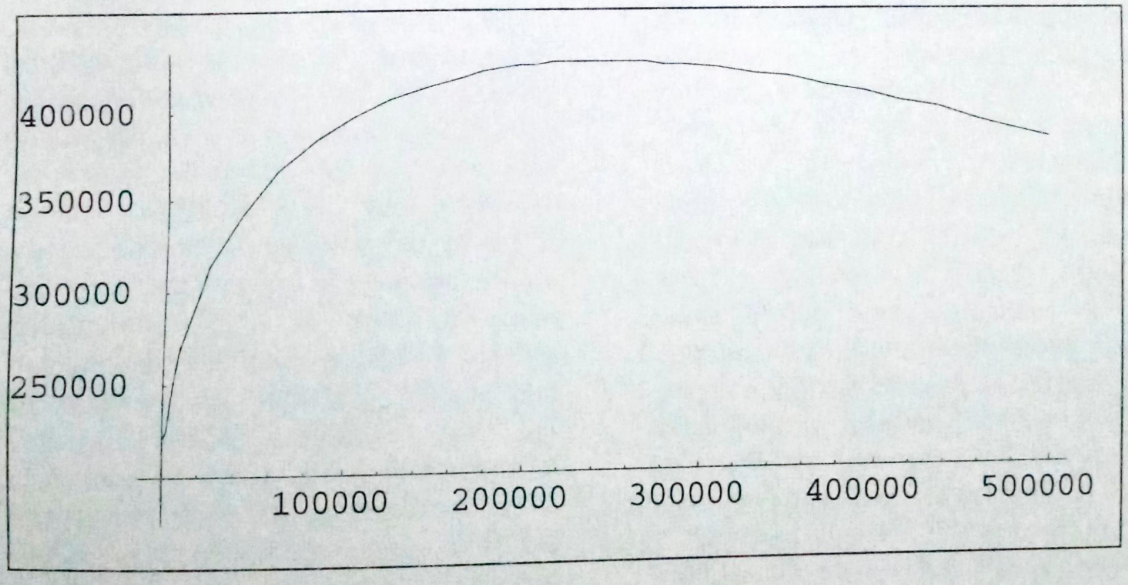


Figure: (b) Profit according to Expenditure for AMPS Group

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