Global solar radiation over Pakistan
Comparison of measured and predicted data

Waheed Hassan Akbari, Firoz Ahmed*, Ravi Shankar Harani

Department of Physics, D. J. Sindh Government Science College, Karachi

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Abstract

The estimates of global solar radiation on horizontal surface at Karachi, Lahore, Multan, Peshawar and Queta were calculated using Angstrom formula which correlates relative global solar irradiance $\overline{H}/\overline{Ho}$ to corresponding relative duration of bright sunshine $\overline{n}/\overline{N_d}$. The coefficient of this relationship were obtained by regression analysis using daily global radiation and sunshine data measured in these cities over a period of eighteen to twenty seven years and twenty seven to thirty four years respectively. The estimated values of global solar radiation $\overline{H}$ were then compared with the measured values both by graphical an statistical methods based on calculation of Mean Bias Errror (MBE), Root Mean Square Error (RMSE), Coefficient of Correlation (r) and Coefficient of Determination (r^2) and were found in good agreement. Thus the correlation of Angstrom is the best estimator for global solar radiation over Pakistan.

Keywords: Solar radiation, statistical evaluation.

Introduction

The design and estimation of the performance of all solar energy conversion system require the knowledge of solar radiation data. These data are not easily available for many countries. As these cannot afford the measurement equipment and techniques involved. Bright sunshine hours are measured at many locations while global radiations are measured at selected stations or locations only. To fill this gap various models have been developed to predict the amount of solar radiation available at any location using weather data or parameters available (1-4). These parameters include (i) the sunshine hours (5-8), (ii) the relative humidity and sunshine (9), (iii) the later two in addition to the mean temperature (10), (iv) the declination angle and the latitude (11-14), (v) the later two in addition to the sunshine duration (15-16), (vi) the number of rainy days, sunshine hours, latitude of location (17), (vii) the sunshine duration, relative humidity, maximum temperature, latitude, altitude and location (18), (viii) the total precipitable water, turbidity and surface albedo (19).

Pakistan is a high insolation country. It receives about 1kW of solar energy for square metre of its landmass for 6-7 hours on the average per day. The number of sunshine hours amount almost to 3000-3300 per year. The weather is most favourable for the utilization of solar energy. The importance of present work stems from the fundamental need of knowledge of solar radiation data in the country.

Of the above mentioned models the most popular and established correlation is the modified form of Angstrom type (20)

$$\overline{H}/\overline{H_0} = a_0 + a_1 (\overline{n}/\overline{N_d}) \ldots \quad (1)$$

Where $\overline{H}$ is the monthly mean daily global radiation falling on a horizontal surface at a particular location ($MJm^{-2} d^{-1}$) $\overline{H_0}$ is the monthly mean daily radiation on a horizontal surface in the absence of any atmosphere at a particular latitude ($MJm^{-2} d^{-1}$) n is the monthly mean daily number of hours of sunshine in a

* Department of Physics, University of Karachi

given month between sunrise and sunset (hr.day^{-1}), and a_0 and a_1 are climatologically determined regression coefficients, $\bar{n}/N_d$ is often called the percentage possible sunshine. This relation is very simple and easy to handle. Furthermore it requires only a few input parameters which are either available from the local observatory or can readily be computed. In the present work equation (1) is used to determine a_0 and a_1 for five cities in Pakistan (Karachi, Lahore, Multan, Peshawar and Queta). The geographical locations of the cities together with the duration of record used are shown in Table 1.

Estimated global radiation

The mean monthly daily global solar radiation, for each of the five cities, was calculated from equation (1) after obtaining the coefficients a_0 and a_1 for each city. In these calculations, the values of $\bar{H}_0$ were calculated from (21) as

$$\bar{H}_0 = 24/\pi \, Isc \, [1+ 0.033.\cos 360\, \bar{n}/365] \, x \, [\cos \Phi \cos \delta \sin \omega s \, 2\pi/360) \sin \Phi \sin \delta \,] \ldots (2)$$

Where Isc is the solar constant (taken as 4.871 MJm^{-2} hr^{-1} in the present calculations), n is the day of the year given for each month (21), Φ is the latitude, δ is the solar declination Table-2 and ω_s is the sunset hour angle given by the equation

Stations	Latitudes (N)		Longitudes(E)		Altitude Metre	Duration of Records	
	deg.	sec.	deg.	sec.		Sunshine Years	Radiation Years
Karachi	24	52	67	08	22.0	33	27
Lahore	31	33	74	20	214.0	34	18
Multan	30	12	71	26	123.0	28	27
Peshawar	34	00	71	31	569.7	34	21
Quetta	30	11	66	57	1799.0	27	27

$$\omega_s = \cos^{-1} (-\tan \Phi \tan \delta).$$

Table 1: Geographical location of stations and duration of records

Month	n for the i^{th} day of the month	For the average day of the month		
		Day of the year		Declination (δ)
		Date	n	Deg.
January	i	17	17	-20.92
February	31+ i	16	47	-12.95
March	59+ i	16	75	-2.42
April	90 + i	15	105	18.79
May	120 + i	15	135	23.09
June	151 + i	11	162	21.18
July	181 + i	17	198	13.45
August	212 + i	16	228	2.22
September	243 + i	15	258	-9.60
October	273 + i	15	288	-18.91
November	304 + i	14	318	-23.05
December	334 + i	10	344	

Table 2: Recommended average day for each month and value of n by months

The coefficients a_0 an a_1 were calculated from values of $\overline{H}/\overline{H_0}$ an $\overline{n}/\overline{N_d}$ for each month of the year from the available ata. The measured values of the monthly mean daily global radiation, H_0, were obtained from the measured daily values. The values of the monthly mean daily extra terrestrial radiation $\overline{H_0}$ were calculated from equation (2) as outlined previously. The measured monthly mean daily number of bright sunshine hours, $\overline{n}$ and the monthly mean daily theoretical number of sunshine hours, N_d were determined from their measured values. Tables 3-7 show results for the five cities.

In this way, the values of $\overline{H}/\overline{H_0}$ an $\overline{n}/\overline{N_d}$ for each month may be determined. Accordingly, twelve equations (one for each month) for each location may be written. Each equation is of form

$$Y = a_0 + a_1 x \quad \ldots\ldots\ldots(3)$$

Where $Y = \overline{H}/\overline{H_0}$ and $x = \overline{n}/\overline{N_d}$. The least square method was then used to calculated the coefficients a_0 and a_1 for the different locations

The following correlations were obtained:

$\overline{H}/\overline{H_0} = 0.345 + 0.369\ (\overline{n}/\overline{N_d})$, Karachi ..... (4)

$\overline{H}/\overline{H_0} = 0.385 + 0.243\ (\overline{n}/\overline{N_d})$, Lahore..... (5)

$\overline{H}/\overline{H_0} = 0.422 + 0.253\ (\overline{n}/\overline{N_d})$, Multan ..... (6)

$\overline{H}/\overline{H_0} = 0.569 + 0.066\ (\overline{n}/\overline{N_d})$, Peshawar ... (7)

$\overline{H}/\overline{H_0} = 0.402 + 0.382\ (\overline{n}/\overline{N_d})$, Quetta... ... (8)

Months	$\overline{H_0}$ (MJm^{-2} d^{-1})	$\overline{n}/\overline{N_d}$	$\overline{H}$ (MJm^{-2} d^{-1})		Error (%)
			Measured	Calculated	
January	23.986	0.8301	15.827	15.622	1.295
February	28.187	0.8165	17.962	18.217	-1.420
March	32.960	0.7550	21.019	20.554	2.212
April	37.076	0.7574	22.484	23.153	-2.975
May	39.292	0.7411	23.447	24.301	-3.642
June	39.941	0.5765	22.735	22.276	2.019
July	39.490	0.3628	19.218	18.911	1.597
August	37.783	0.3848	17.837	18.399	-3.151
September	34.338	0.6039	19.846	19.498	1.754
October	29.501	0.8121	19.344	19.018	1.685
November	24.921	0.8570	16.413	16.479	-0.402
December	22.726	0.8300	14.864	14.801	0.424

Table 3: Comparison between measured and calculated values of $\overline{H}$ for Karachi

Months	$\overline{H_0}$ (MJm^{-2} d^{-1})	$\overline{n}/\overline{N_d}$	$\overline{H}$ (MJm^{-2} d^{-1})		Error (%)
			Measured	Calculated	
January	20.131	0.6837	10.719	11.0835	-3.405
February	24.894	0.7160	13.942	13.8993	0.308
March	30.643	0.6688	18.213	16.7603	7.978
April	36.110	0.7226	20.893	20.2219	3.212
May	39.543	0.7346	22.819	22.2595	2.450
June	40.784	0.6539	22.317	22.1595	0.703
July	40.065	0.5524	20.558	20.7822	-1.089
August	37.332	0.5886	18.758	19.6925	-4.985
September	32.544	0.7256	18.255	18.2486	0.033
October	26.521	0.8319	15.115	15.5552	-2.911
November	21.215	0.8403	12.435	12.4864	-0.410
December	18.764	0.7329	10.216	10.5549	-3.318

Table 4: Comparison between measured and calculated values of $\overline{H}$ for Lahore

Months	$\overline{H_0}$ (MJm-2 d^{-1})	$\overline{n}/\overline{N_d}$	$\overline{H}$ (MJm^{-2} d^{-1})		Error (%)
			Measured	Calculated	
January	20.894	0.7128	12.687	12.5876	0.7803
February	25.556	0.7005	15.408	15.3162	0.5971
March	31.126	0.7005	18.758	18.6541	0.5544
April	36.335	0.7138	22.735	21.8983	3.6815
May	39.530	0.6976	24.201	23.6621	2.2272
June	40.654	0.6631	23.531	23.9807	-1.9124
July	39.988	0.6404	23.196	23.3586	-0.7027
August	37.456	0.6825	21.731	22.2778	-2.5171
September	32.928	0.7442	20.391	20.0978	1.4369
October	27.127	0.8076	16.748	16.9853	-1.4151
November	21.951	0.8197	13.859	13.8165	0.3031
December	19.544	0.7506	11.556	11.9604	-3.4960

Table 5: Comparison between measured and calculated values of $\overline{H}$ for Multan

Months	$\overline{H_0}$ (MJm^{-2} d^{-1})	$\overline{n}/\overline{Nd}$	$\overline{H}$ (MJm^{-2} d^{-1})		Error (%)
			Measured	Calculated	
January	18.648	0.6278	11.263	11.3799	-1.0388
February	23.583	0.6354	14.655	14.4032	1.7195
March	29.670	0.5417	18.297	17.9382	1.9621
April	35.628	0.6127	21.731	21.7065	0.1104
May	39.521	0.6841	25.122	24.2637	3.4135
June	40.990	0.7028	25.959	25.2160	2.8622
July	40.168	0.6429	23.447	24.5522	-4.7128
August	37.043	0.6598	21.354	22.6832	-6.2237
September	31.759	0.6905	19.386	19.5116	-0.6499
October	25.320	0.7724	15.869	15.6920	1.1154
November	19.778	0.7729	12.728	12.2580	3.6926
December	17.253	0.8604	10.468	10.7922	-3.0951

Table 6: Comparison between measured and calculated values of $\overline{H}$ for Peshwar

Months	$\overline{H_0}$ ($MJm^{-2} d^{-1}$)	$\overline{n}/\overline{N_d}$	$\overline{H}$ ($MJm^{-2} d^{-1}$)		Error (%)
			Measured	Calculated	
January	20.894	0.6691	14.110	13.7358	2.6293
February	25.556	0.6930	16.581	17.0376	-2.7562
March	31.126	0.6495	20.349	20.1805	0.8256
April	36.335	0.7094	20.950	24.3899	-1.8372
May	39.530	0.8043	27.467	27.9690	-1.8276
June	40.654	0.8304	29.686	29.1700	1.7382
July	39.988	0.7590	26.881	27.6004	-2.6748
August	37.456	0.8485	26.420	27.1346	-2.7063
September	32.928	0.7974	24.201	23.2110	4.0907
October	27.127	0.8938	20.349	20.1218	1.1155
November	21.951	0.8649	16.162	16.0398	0.7549
December	19.544	0.7308	13.608	13.2789	2.4177

Table 7: Comparison between measured and calculated values of $\overline{H}$ for Queta

Results and discussion

The present results indicate that the coefficients a_0 and a_1 depend on location. Comparison between estimated and measured global radiations shows that the present equations (4-8) can be used to predict the monthly mean daily global radiation in the locations considered in this study.

The percentage errors between the calculated and measured values of the monthly mean daily global radiation $\overline{H}$ were estimated for the five cities for the twelve months of the year. These erros are given in Tables 3-7. The minimum and maximum percent error for the five cities ranged as under.

Through the monthly average of the measured global radiation on a horizontal surface and the average monthly total hours of sunshine per day has been obtained for the period of 18 to 27 years and for the periods of 27 to 34 years respectively comparison of an individual calculated value against a measured or percent difference is inadequate to test the accuracy of a predictive model.

Figures 1-5 show the plot of the predicted values along with the measured values. It appears from the figures that a visual comparison of these results is also insufficient to test the accuracy of the predictive model.

Station	Minimum percent error	Maximum percent error
Karachi	-3.6420	2.2120
Lahore	-4.9850	7.9780
Multan	-3.4960	3.6815
Peshawar	-6.2237	3.6926
Quetta	-2.7562	4.0907

Figures 6-10 show a relationship between the predictive global solar radiation and measured global solar radiation. It appears from the graphs that there is fairly good agreement between the measured monthly values of global solar radiation and the predicted values obtained with the modified Angstrom Correlation.

To check further the accuracy of the predictive model a number of useful statistical tests based on calculation of Mean Bias Error (MBE), Root Mean Square Error (RMSE), Correlation Coefficient (CC or r) and Coefficient of Determination (r^2) are made. Results of these tests are listed in Table 8. From Table 8 it is evident that the MBE ranged between –0.0446 to + 0.0194 which is very close to the ideal value i.e. zero, whereas the RMSE ranged between 0.3932 to 0.6361 which is not close to zero. The values of Correlation Coefficient ranged between 0.9884 to 0.9960. The Coefficient of Determination ranged between 0.9770 to 0.9920. The minimum and maximum for both the cases are very close to unity. Thus correlation of Angstrom in my opinion is the best estimator of global solar radiation.

Acknowledgements

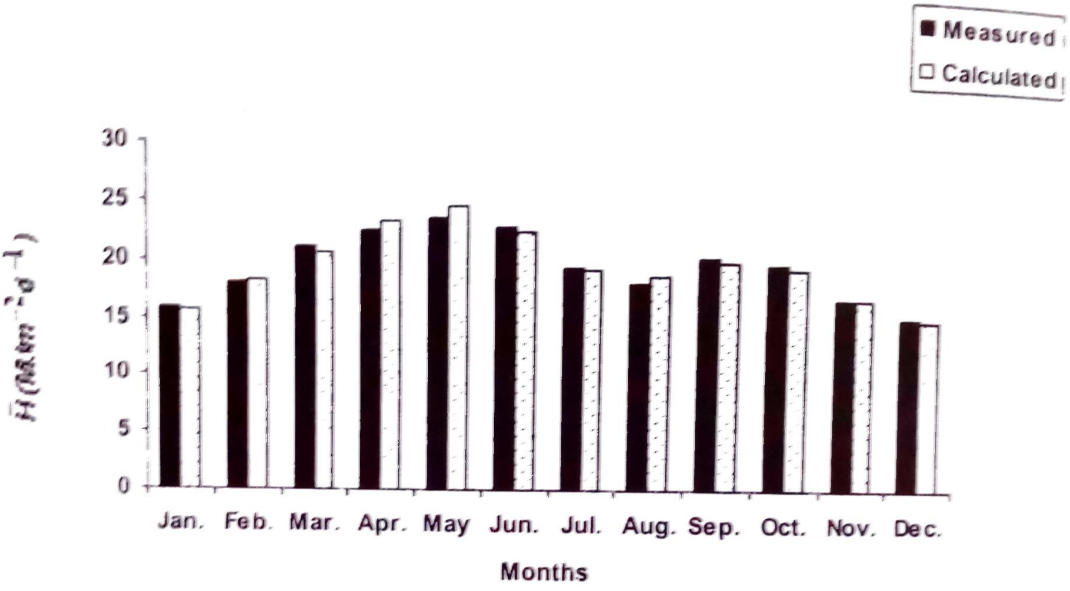
The authors are thankful to the Pakistan Meteorological Department Computerized Processing Center (CDPC), University Road, Karachi for providing the requisite data.

Table 8 also shows that the sum ($a_0 + a_1$) which represents that clear day fraction of $\overline{H_0}$ decreases systematically with the increase in Latitude except for Queta where the sum ($a_0 + a_1$) increases abruptly though the latitude lies between 24052′ and 30012′. The annual average of $\overline{n/N_d}$ also tends to show a similar variation with Latitudes.

Station	Lititude(N)		Annual Average		Regression coefficients			MBE	RMSE	Correlation Coefficient	Coefficient of Determination
	Deg.	Sec.	$\overline{H/H_o}$	$\overline{n/Nd}$	a_0	a_1	a_0+a_1			CC or r	r^2
Karachi	24	52	0.601	0.694	0.345	0.369	0.714	0.0194	0.4430	0.9884	0.9770
Lahore	31	33	0.555	0.704	0.385	0.243	0.628	-0.0446	0.5965	0.9908	0.9817
Multan	30	12	0.604	0.719	0.422	0.253	0.675	-0.0171	0.3932	0.9960	0.9920
Peshawar	34	00	0.614	0.684	0.569	0.066	0.635	0.0098	0.6361	0.9923	0.9847
Quetta	30	11	0.696	0.771	0.402	0.382	0.784	0.0092	0.5220	0.9958	0.9916

Table 8: MBE, RMSE. Regression coefficients, correlation coefficient and coefficient of determination.

FIG. 1: COMPARISON BETWEEN MEASURED AND CALCULATED VALUES OF $\bar{H}$ FOR KARACHI



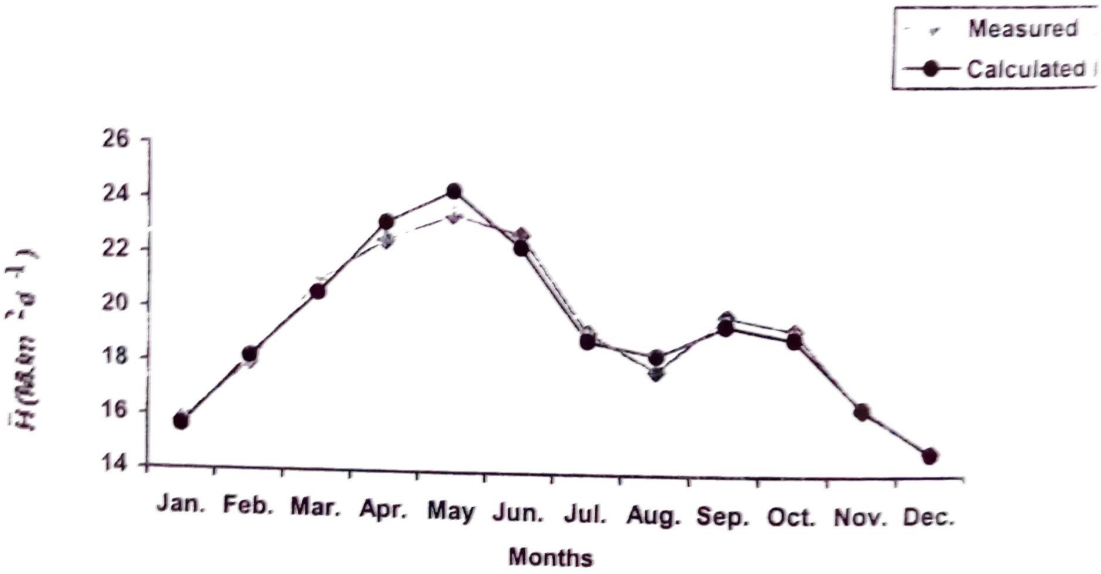


FIG 2: COMPARISON BETWEEN MEASURED AND CALCULATED VALUES OF $\bar{H}$ FOR LAHORE

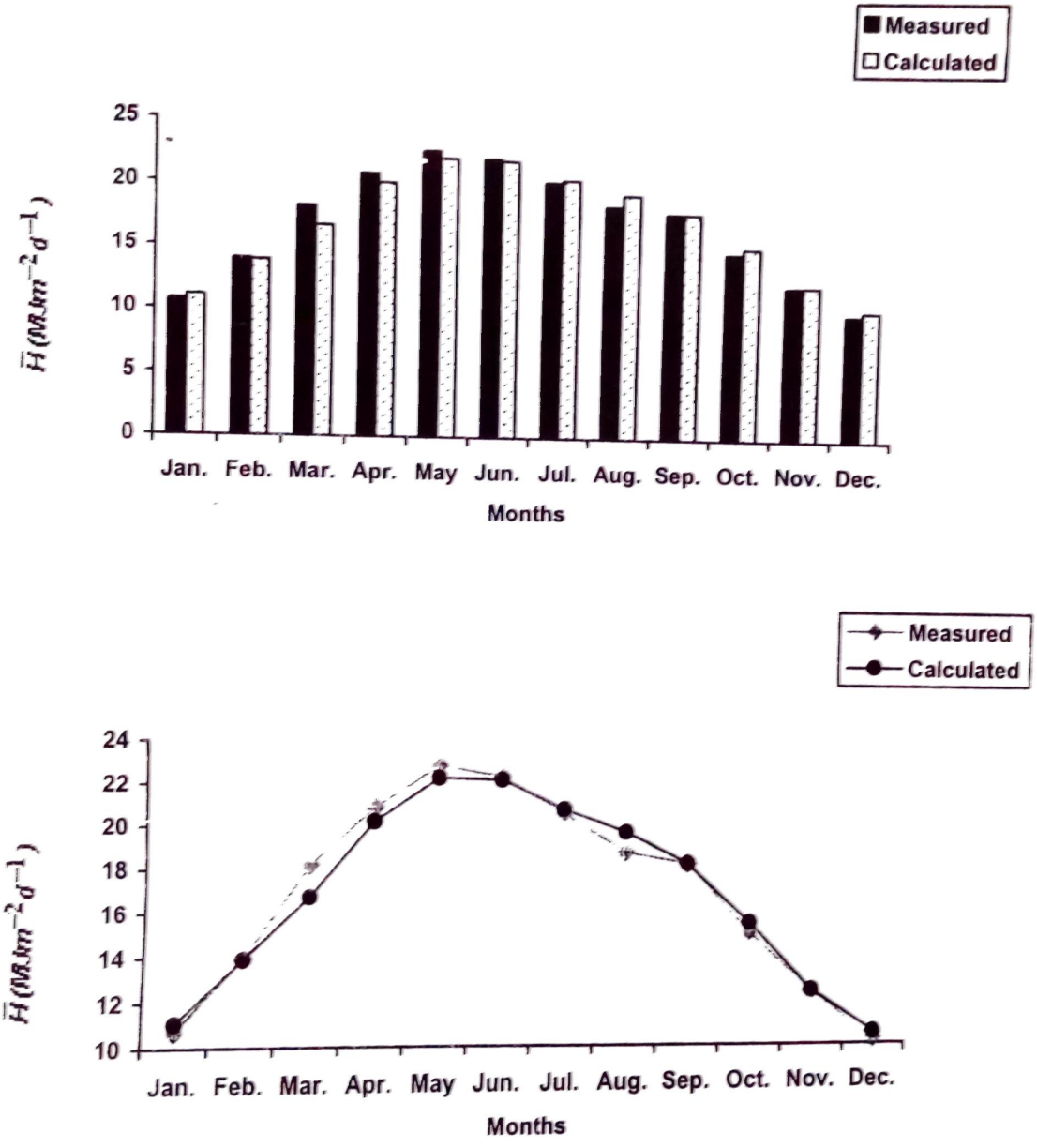
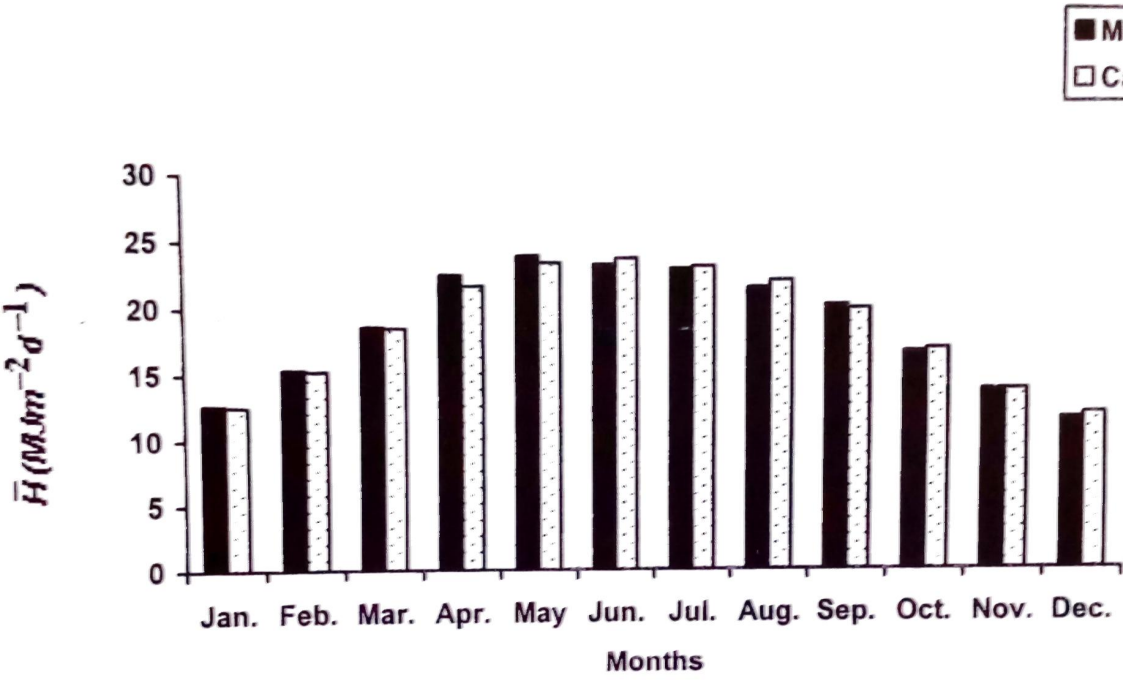


FIG 3: COMPARISON BETWEEN MEASURED AND CALCULATED VALUES OF $\bar{H}$ FOR MULTAN



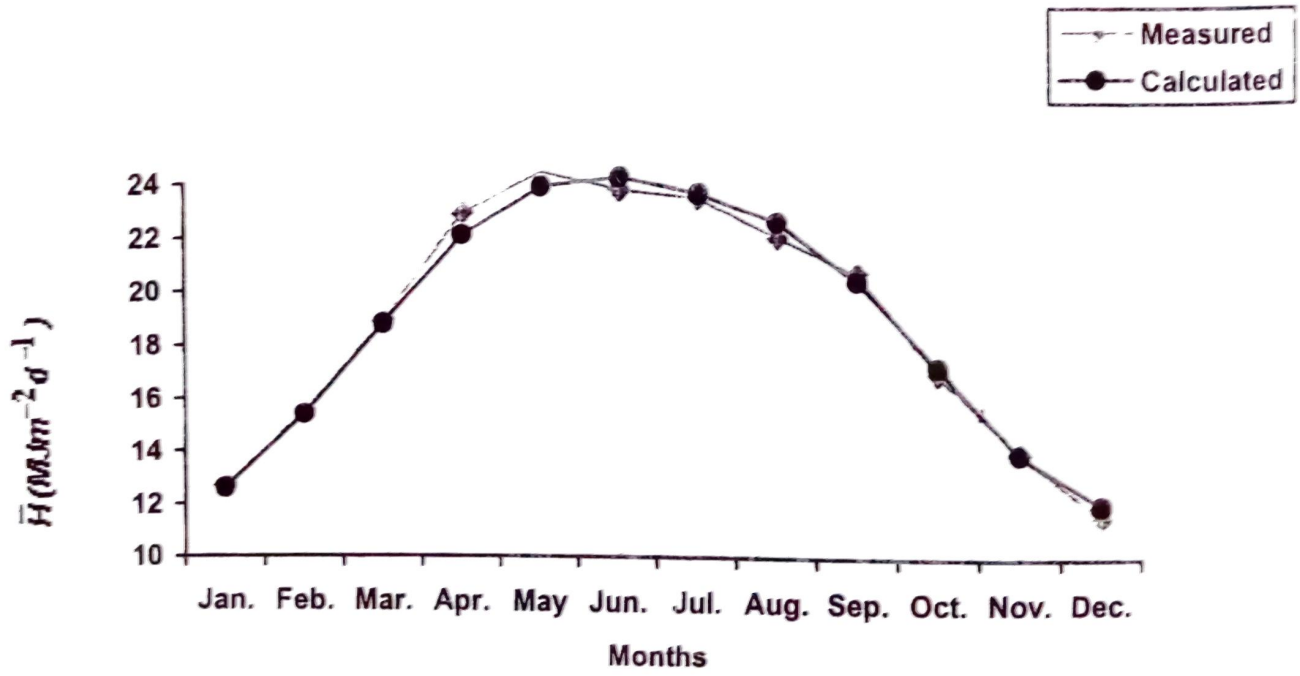


FIG 4: COMPARISON BETWEEN MEASURED AND CALCULATED VALUES OF $\bar{H}$ FOR PESHAWAR

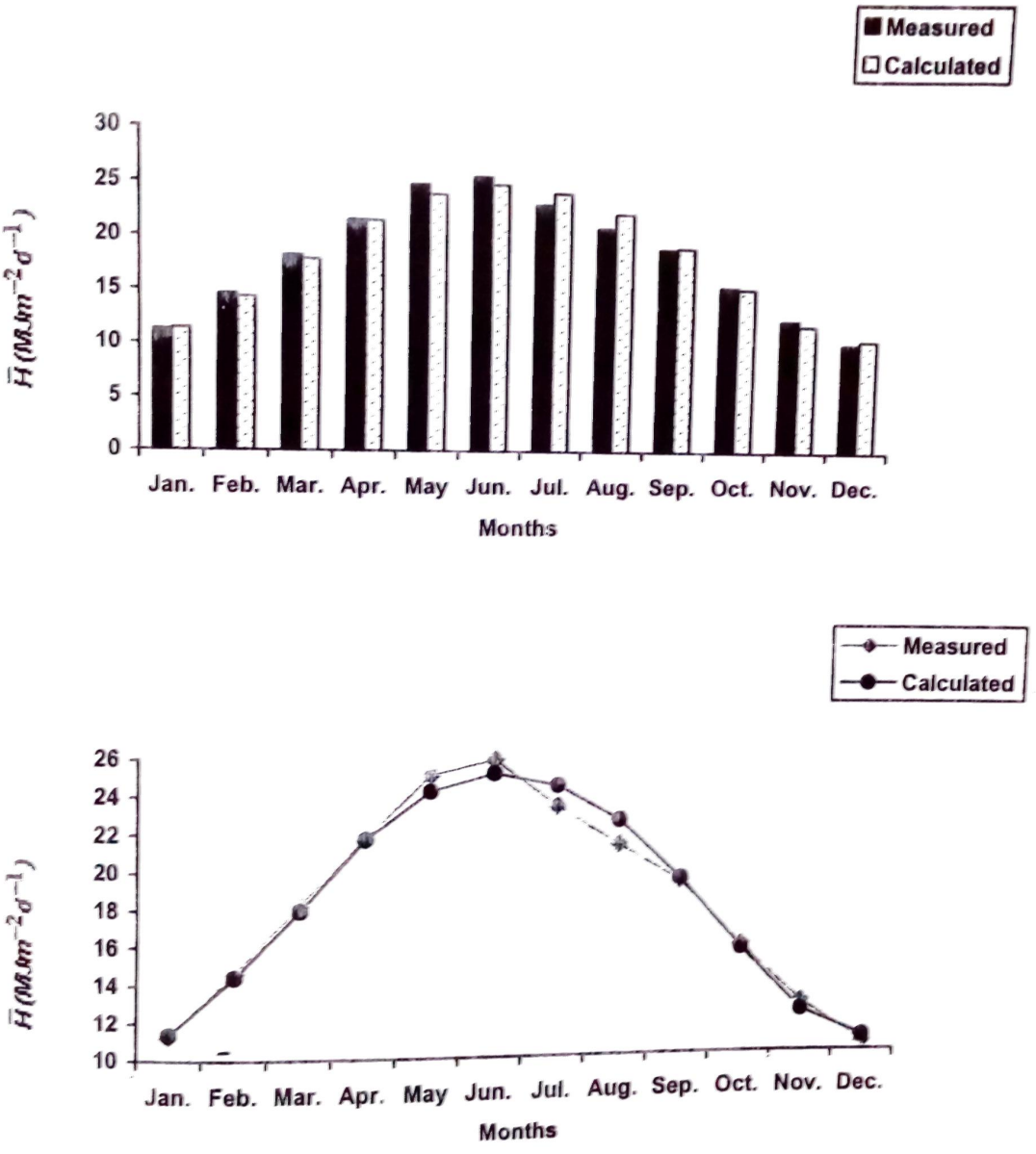


FIG 5: COMPARISON BETWEEN MEASURED AND CALCULATED VALUES OF $\bar{H}$ FOR QUETTA

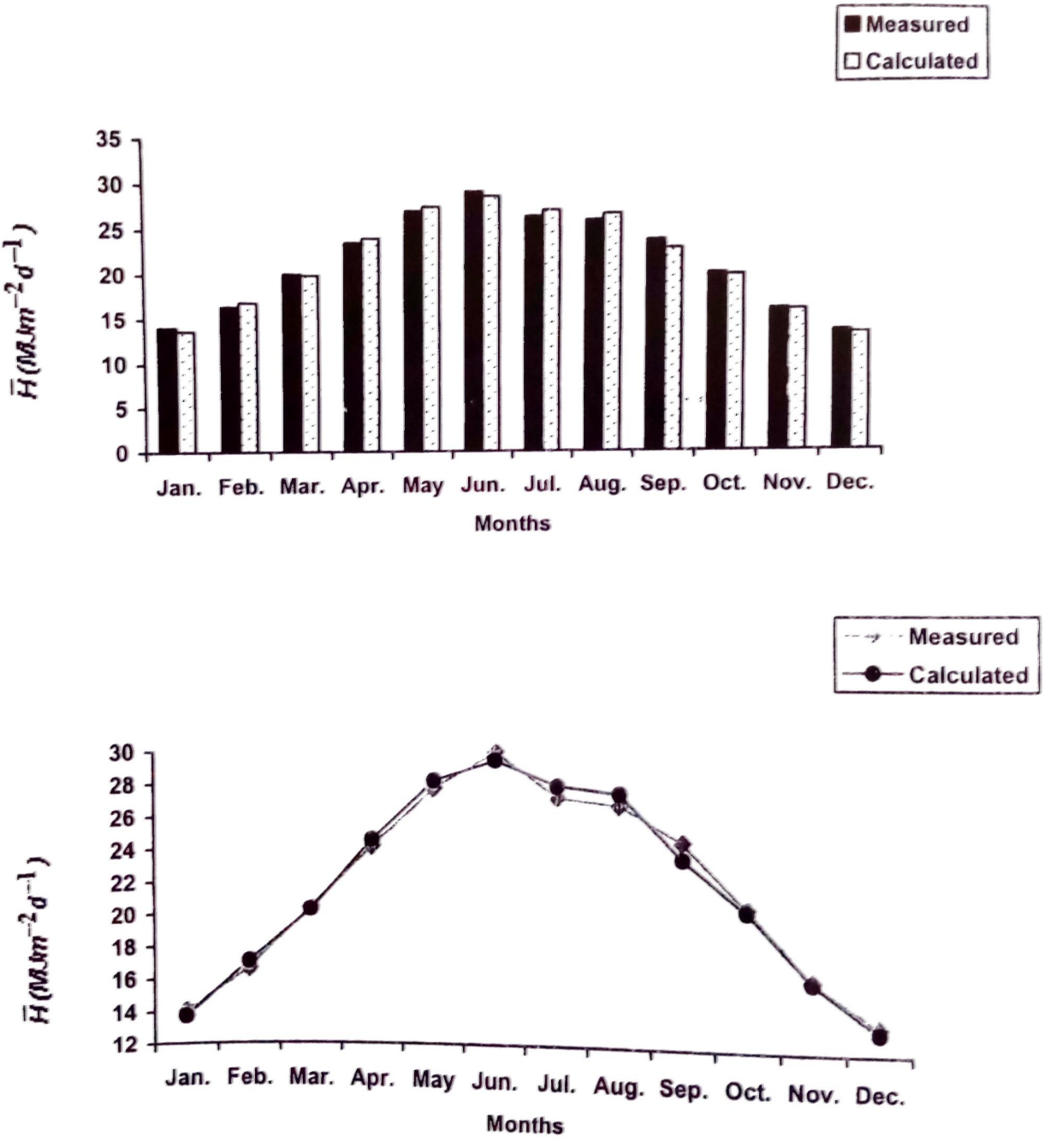


FIG 6: RELATIONSHIP BETWEEN THE CALCULATED AND MEASURED $\overline{H}(MJm^{-2}d^{-1})$ FOR KARACHI

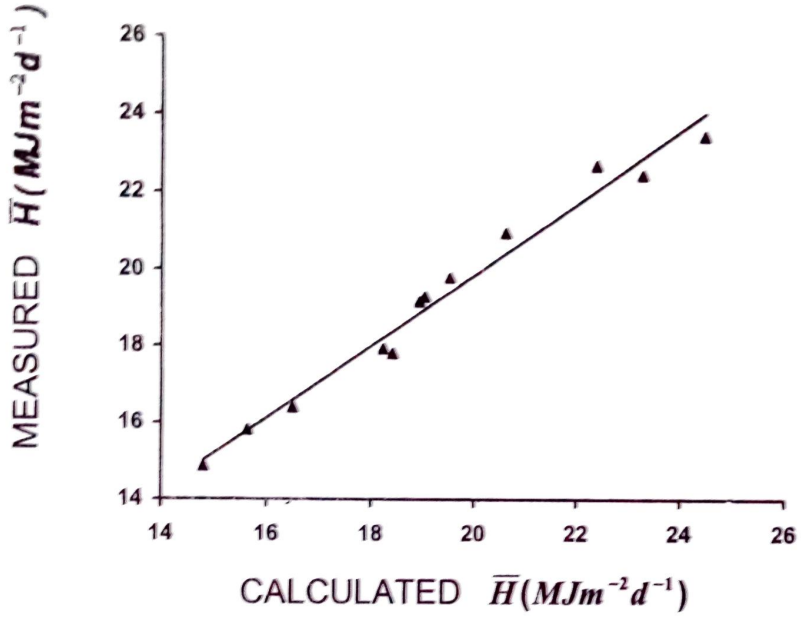


FIG 7: RELATIONSHIP BETWEEN THE CALCULATED AND MEASURED $\overline{H}(MJm^{-2}d^{-1})$ FOR LAHORE

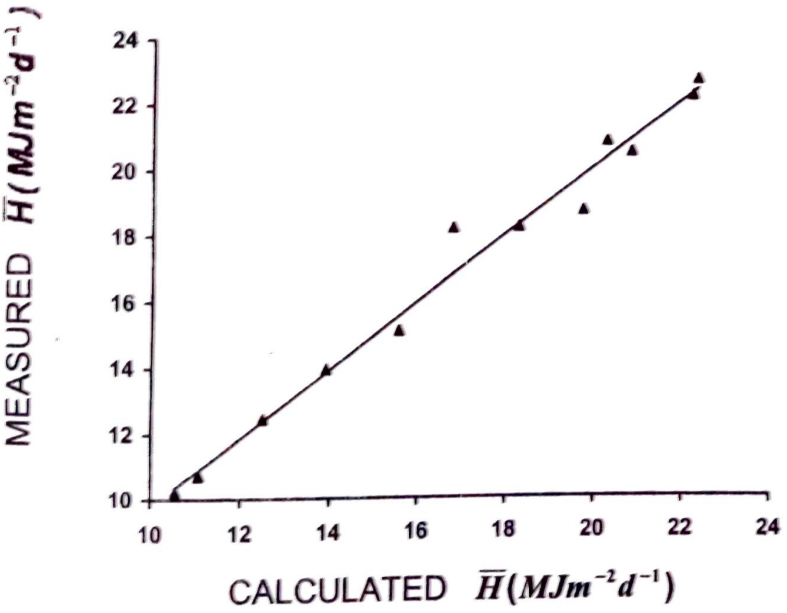


FIG 8: RELATIONSHIP BETWEEN THE CALCULATED AND MEASURED $\bar{H}(MJm^{-2}d^{-1})$ FOR MULTAN

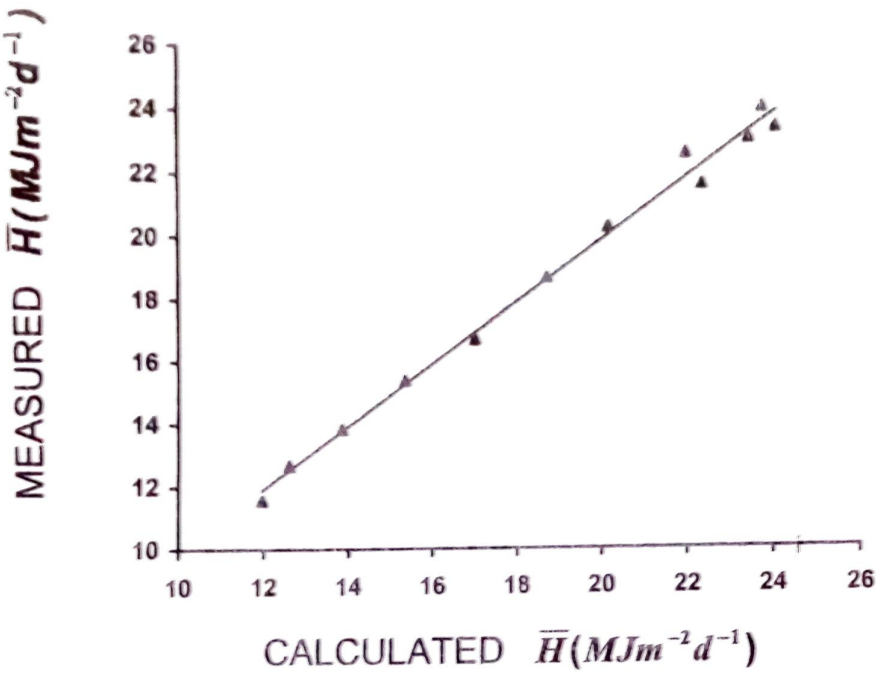


FIG 9: RELATIONSHIP BETWEEN THE CALCULATED AND MEASURED $\bar{H}(MJm^{-2}d^{-1})$ FOR PESHAWAR

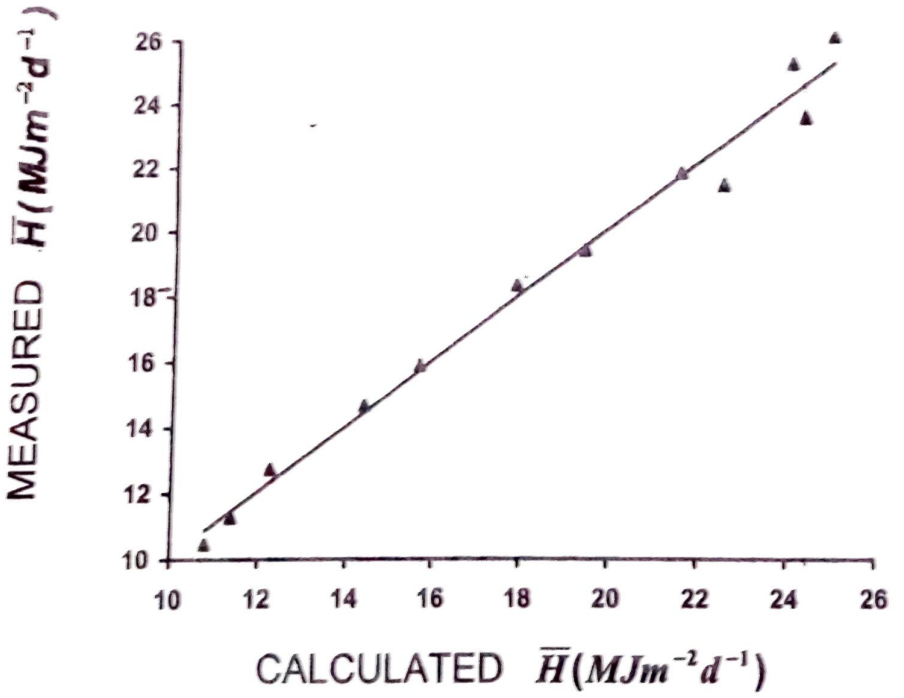
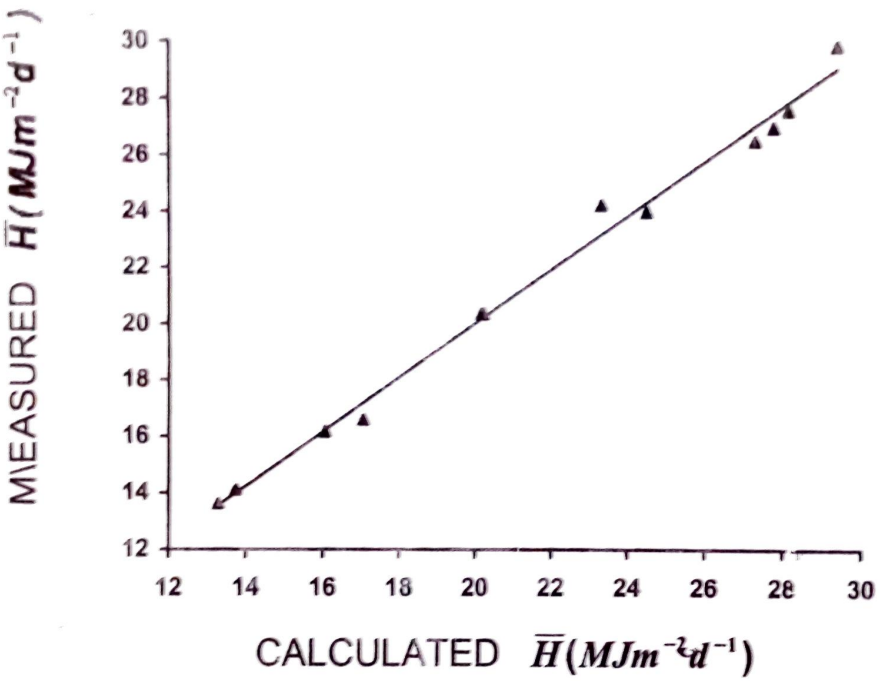


FIG 10: RELATIONSHIP BETWEEN THE CALCJLATED AND MEASURED $\overline{H}(MJm^{-2}d^{-1})$ FOR QUETTA



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