Sinh Univ. Res. Jour. (Sci. Ser.) Vol.: 37 (1) 41 -46 (2005)

# Mathematical Model for Evaluating mass of Mangroves Forest along Coast Line of Karachi/Indus Delta

N. Khan, M. Hussain, T. S. Rehmani

Department of Mathematics, University of Karachi, Karachi -75270, Pakistan

## Abstract

Mangroves all over the world grow in tropical region, along tidal estuaries in salt marshes and muddy coast.In Pakistan, the mangrove forest spreads over the coastline of Sindh and Baluchistan. The area of mangroves has been estimated at about 250,000 ha, ranking the fifth largest mangrove area in the world. In this paper an effort has been made to develop a mathematical model, which can be appropriate to determine the mass of mangroves. To achieve this objective we require two parameters i.e. volume and density of the mangroves. In order to calculate the volume, the shape of the plant may be considered like mushroom. The leaves/branches scattered round the stem in the from of semi sphere and the exposed portion of stem in the from of cylinder.

**Keywords:** Mathematical model, Mangroves, Karachi coast, Indus delta.

## Introduction

Mangrove is a woody plant community which is found between the sea and land in areas, which are inundated by tides. Various species of mangroves share the ability to live in salt water.

The richest mangrove communities occur in tropical and sub-tropical areas where the water temperature is greater than 240C in the warmest month, where the annual rainfall exceeds 125mm and mountain ranges greater than 700m high are found close to the coast. The proximity of mountains tends to ensure the rainfall. In addition, they need protection from high-energy waves, which can erode the shore and prevent seedlings from becoming established.

Mangroves exist in a constantly changing environment. Daily the sea inundates the community with salt water while, at low tide, especially during periods of high rainfall, it may be exposed to floods of fresh water. Apart from suddenly altering the salinity levels, these fluctuations in water can alter temperatures as well. Different mangrove species have different requirements. Some are more tolerant of salt than others. Other factors, which affect their distribution, include wave energy, oxygen levels, drainage and differing nutrient levels. Where one species finds its preferred conditions or at least those, which it is able to tolerate better than other plants, it tends to become dominant. This has led to distinct zones among mangroves.

The Indus delta stretches over an area of 600,000 hectares between Karachi and Southwestern border of India. It is a typical fan shaped delta built up by discharge of large quantities of silt washed down from the Karakoram and Himalayan mountain ranges by the river Indus. It houses the world's largest arid climate mangrove ecosystem consisting of seventeen major creeks and extensive mud flats, sand dunes, salt marshes and mangroves (158, 500ha). The Indus delta mangroves are perhaps unique in being the largest area of arid climate mangroves in the world. An estimated 135,000 people are depending on the resources of this ecosystem for their livelihood, and it supports a variety of lives.

Most of Pakistan's commercial marine fishery operates on the coast of Sindh, where fishing occurs within the mangrove creeks and in the neritic waters off the mouths of creeks. The marine fishery of Pakistan relies greatly on shrimps, being the most important of all the categories in terms of value of landing. This lucrative foreign exchange earning commodity has highly influenced the development of fishing crafts and gears in the country especially along the coast of Sindh, and this development during the past three decades was predominantly for catching more and more shrimps . It was estimated (in 1997) that the value of shrimp caught from the coastal waters of Sindh was around Rs. 2,834 million, while the value of mangrove dependent finfish during the same year was Rs. 780 million.

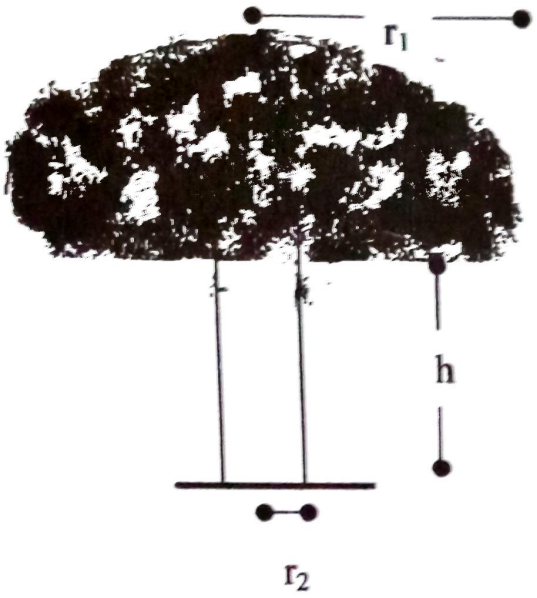
In order to delineate the importance of mangroves to the rich marine fishery of the Indus delta, it will be better for us to first review some important oceanographic features of the continental shelf of Pakistan. The northern Arabian Sea is believed to have high productivity and this consideration leads to the optimism that the off-shore marine fishery resources of Pakistan may be of considerable magnitude. However there are certain hydrographic features in off-shore waters which do not seem to encourage large fishery production, at least affect it in a negative way. For instance the continental shelf of Balochistan is narrow (13-32km wide), anoxic conditions resulting in mortality of fish and shrimps or their migrations are also known to occur. In the Arabian Sea, everywhere north of 200 N, a layer of extremely low oxygen concentration extends from above 200m down to more than 1200m depths.

Besides the tangible products, the Indus delta mangroves protect the coastal villages from tidal and wave actions especially during monsoon season, also it is claimed that the mangroves of the Port Qasim area along the creeks have resulted in a much-reduced dredging cost. Different species of shell and finfishes use the mangrove ecosystem as feeding, breeding and nursery ground. It also provides an excellent habitat for snakes birds and mammals. Migratory birds find shelter here during winter. Some of the most common animals cited by villagers include pelicans, flamingos, kites, herons, egrets, jackals.

As illustrated earlier and having learned the lessons that mangroves are the principal hidden causative factor for higher fishery productivity in the coastal waters of Sindh. Unfortunately the mangrove vegetation cover of the Indus delta faces the problem of degeneration; it decreased from 263,000 ha in 1977 to about 160,000 ha in 1991. In this connection several man-made causes have been identified including camel browsing, fodder collection, fuel wood collection. The fishery resources are also facing the problem of degeneration as a result of environmental degradation and exploitation pressure. Thus, the conservation strategy objective should not only include management of mangrove plant resources but also regulation of the present virtually uncontrolled fishery as well, operating in the different habitat zones of this ecosystem in order to have a sustainable production output in future.

## Mathematical Modeling

To develop the mathematical model, which will be appropriate to determine mass of the mangroves, two parameters i.e.: volume and density are used. The shapes of the plant as mentioned in Fig. 1 is considered as a semi sphere (for leaves/ bushes) and a cylinder (for exposed portion of stem). The volume of these shapes are obtained below as $V_1$ and $V_2$ respectively, which after addition is multiplied with the density of mangroves to obtain mass of the mangroves.



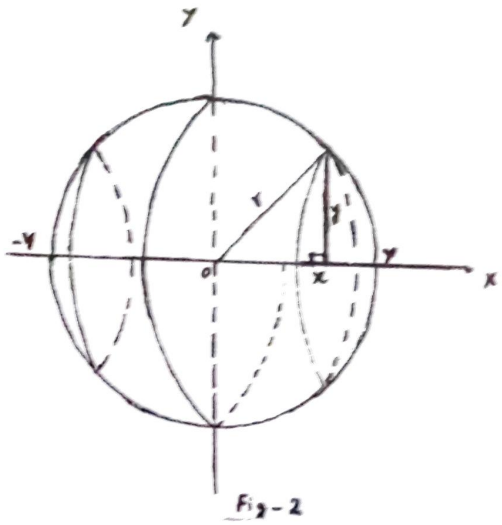




Fig-2

**Fig.1. Shape of the plant and geometrical model**

Consider the sphere whose center is at the origin then the plane 'px' intersects the sphere in a circle whose radius is 'y' also shown at Fig-2. Then by Pythagoras theorem:

$Y = \sqrt{(r^2 - x^2)}$

Therefore the crossectional area $A(x) = \pi y^2$

$A(x) = = \pi (r^2 - x^2)$

Using the formula of volume i.e:

$V = {}_a\int^b A(x)\, dx$ here a = -r and b= r

Or $V = {}_{-r}\int^{r} A(x)\, x$

$= {}_{-r}\int^{r} \pi (r^2 - x^2)\, dx$

The above integral gives the volume of entire sphere whereas we are interested in the volume of semi sphere. Then the limits for integration will be o and r

$\text{Volume} = \pi\, {}_o\int^{r} (r^2 - x^2)\, dx = \pi \left| r^2 x - x^3/3 \right|_o^r$

$= \{ r^3 - r^3/3 \} = \pi (2r^3/3)$

volume of the semi sphere is

$V_1 = 2\pi r_1^3/3$ (1)

The stem of the tree is normally of cylindrical shape an as such its volume will be:

$V_2 = \pi r_2^2 h$ (2)

The total volume will be (from equation 1 and 2 i.e: $V_1$ $V_2$)

$2/3\pi r_1^3 + \pi r_2^2 h$ or $\pi (2/3\pi r_1^3 + \pi r_2^2 h)$

The mass of mangrove tree is given by v x d, where 'd' is density of the mangrove

Mass = {volume of the semi sphere ($V_1$) + volume of the stem of tree ($V_2$)}x density of mangroves

$$\text{Mass} = \pi (2/3 \pi r_1^3 + \pi r_2^2 h) x\, d \qquad (3)$$

as

$r_1$ = the radius of the mushroom of leaves / branches scattered around the stem of a mangrove tree.

$r_2$ = the radius of the stem exposed

h = height of the stem exposed

d = density of mangroves

In order to observe the validity of the model a survey of estuaries around coastal areas of Karachi have been carried out to obtain the required data. The different age groups / sizes of the mangroves plant were selected for the purpose; the number of plants in a specified area was also counted.

The detail of data / dimensions (on average basis) in respect of mangroves trees has been taken and are as follows:

Circumference of the stem: 77cms, gives radius = 12.25 cms = $r_1$

Length of the exposed portion of stem: 272 cms = h

Radius of canopy ( leaves / bushes ): 420 cms = $r_2$

Number of mangroves trees per 10 x 10m = 37

Density of mangroves = d 3 kg / $m^3$

Substituting the above values of $r_1$, $r_2$, h and d in equation (3) , we get mass of an average mangrove tree i.e:

Mass = $M_a$ = 452 kgs

As the average number of tree in 10 x 10m area = 100m 2 = $N_t$ = 37.

Therefore, the total mass of mangroves tree in this area = $M_a$ $N_t$ = 16724 kgs.

According to the recent information obtained through National Institute of Oceanography and different other sources, the area of mangroves along the coastal areas of Karachi has been estimated about 250,00 ha.

Therefore, the total mass of mangroves for the above mentioned area (as 1 hectare = $10^4$ $m^2$ and 1 metric ton = $10^3$kgs) becomes: 41.81 x $10^7$ metric tons.

**Results and Discussion**

In order to get best possible results, the mangroves trees have been divided into three categories:

Categories 1–Fully grown up trees

Categories 2 – Middle aged trees

Categories 3 – Young trees

The number of trees in the specified area ($100m^2$) has been counted, different dimensions required for the model noted and the average values of parameters taken which are used to obtain the desired objective. To verify the validity of the model a small tree has been cut and its mass was found by traditional methods. The comparison of both the masses revealed that the mass calculated through model was slightly more than the mass obtained through later procedure. The reason for the small deviation is because the mathematical model gives the mass considering the mangroves tree a complete solid semi sphere having leaves/branches. Whereas the second method gives only the mass of the leaves/branches. As the newly born and other small trees have not been taken into account the mass of these left over baby trees shall automatically be adjusted in the difference.

## Conclusions

The model has been tested and found feasible for evaluating the mass of mangroves forest of any dimension. It is pertinent to mention here that the mangroves plant living in the hyper saline environment has slightly more mass than a plant in a lesser saline environment. As such in the next paper due consideration shall be given to this aspect for achieving better results.

## References

Giordano, F. R. Weir, M. D. Finney, J. (2003) Thomas' CALCULUS, Pearson Education Incl, New York. Pp. 32-185.

United Nations Publication: Economic and Social Commission for Asia and Pacific. (1990). Coastal Environmental Management Plan for Pakistan. Pp. 78-84.

Kreyszig, E., (1983) Advanced Engineering Mathematics. John Wiley, New York. Pp 358-393.

Thiel, A. (1997) Reef Aquarium Farming News # 12. Geothermal Aquaculture Research, USA. Pp. 1-3.