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Predicition of Floods of Indus River at Guddu Barrage by Fitting Gumbel and Generalized Extreme Value Probability Distributions

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Abstract

Gumbel and Generalized Extreme Value (GEV) distributions are fitted on 38 years (1962-1999) maximum flood series for river Indus at Guddu barrage, Pakistan. The distribution parameters are estimated by Method of Moments (MOM), Maximum Likelihood Method (MLM) and Power Weighted Moment (PWM). Gumbel distribution appears to be appropriate for modelling the flood data on the basis of Goodness - of - fit tests, Standard Errors (S.E's) of Quantile estimates coefficients, of Skewness (C_s), Kurtosis (C_k) and sample L-Moment ratios and MLM is the most efficient method. The flood estimates at various return periods suggest that authorities should take appropriate measures for safety of the barrage as 100-years flood prediction at Guddu is 1.5 million cusecs, which is much higher than its existing capacity of 1.1 million cusecs.

Keywords: Indus river, Floods, Guddu barrage, Probability distribution.

Introduction

The barrage at Guddu, completed in 1962, was designed to pass a minimum flood discharge of 1.1 million cusecs. In 1976 it came under maximum pressure to pass flood discharge of 1.2 million cusecs which was 0.1 million casecs above its designed capacity. In 1976, 1986 and 1988 more than 1.15 million cusecs passed through Guddu Barrage. Hence, there is a possibility of damage to the structure.

Bhutto and Shaikh (1987) analyzed the data of 24 annual flood peaks at Guddu, from 1962 to 1985. They developed Foster Type-1 and 3, Hazen and Gumbels extreme flood frequency curves. Their estimates for 100-year floods at Guddu were 1.45, 1.45, 1.47 and 1.40 million cusecs, respectively by each curve. Hence, there is a need of extensive analysis for the larger sample of peak floods.

As the exact sequence of peak flow for the future years can not be predicted, we have fitted Gumbels and GEV distributions on the data. The choice of Probability distribution with the help of Goodness of Fit tests does not necessarily lead to a unique distribution. However, C_s, C_k and L-moments ratios give some idea about the parent probability distribution (Chowdhury 1995), (Iddress, 1994) on flood data of Jehlum River at Rasul, Memon and Shaikh (2005, a, b) on Indus River at Kotri and Sukkur and are also recommended by World Metrological Organization (WMO). In U.K. and several other countries, GEV is used (Cunnane, 1989).

In Bangladesh different organizations employ different flood frequency distribution functions in the analysis, design and planning studies. The Bangladesh Water Development Board (BWDB) and several other organizations employ EVI which is also known as Gumbel distribution. In the Flood action plan (FAP), consultants are using GEV (Chowdhury,1995) . Ahmed *et al* (1988) compared the Log – Logistic distribution to GEV,LNand P (Bund Manual,1977) distributions by using data from Scotland. It was found to perform better then

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other distribution. Lowery and Nash (1970) and Singh (1986) compared a number of methods of fitting EVI 2 distribution to the sample data. MOM was found to be most accurate, next to MLM. MOM was also found to be virtually unbiased and the simplest to apply. Landwehr *et al.* (1979) developed estimation of parameters of Gumbel's distribution by PWM. The PWM estimates were comparable to other estimates. Cunnane (1989) has presented an excellent review of the issue of selecting a distribution for a region or country. He observed that χ^2 and S-K tests are of little value in this context and recommended EVI. Abdul Sabur (1982) performed the regional flood frequency analysis of the annual flood peaks of Thailand. Gumbel, LN , P were selected. Based upon χ^2 –test, S-K test and other considerations, the Gumbel distribution was the best.

Yang (1987) compared Normal, L N 3, P 3 and Gumbel distribution using annual maximum (AM) water levels and discharges obtained from hydraulic computations at selected points along the Tansui river. Chowdhury and Karim (1995) compared Gumbel, LN , LN GEV, P and LP using the data of a river in Bangladesh. On the basis of Probability Plot Correlation Coefficient (PPCC) and Root Mean Square Error (RMSE), the performance of GEV is the best also in the case of AM discharge data followed by LN Gumbel and LN. Haktanir (1991) fitted nine distributions with the data from 45 unregulated streams in Turkey and concluded that LN and EVI were superior to other distributions. Onoz and Bayazit (1995) used seven distributions on the data from 19 stations in the world and found that GEV was superior to other distributions. Fatih (2002) made statistical comparision of currently popular probability models. The Gumbel, Log-Logistic, P, LP and LN and distributions were applied to the series of annual instantaneous flood peaks for 13 flow gauging stations in the Seyhan basin in Turkey. Gumbel using MOM for flow stations in the Seyhan river basin were found to be the best models.

Present data is collected from the office of Chief Engineer, Guddu Barrage, at Sukkur and is presented in Fig.1. The histogram and frequency curve of flood peaks at Guddu are presented in Fig.2. The values of statistic such as Mean, Median, S. D. for 38 years are calculated and presented in Table:1, along with those obtained for 24 years (Bhutto and Shaikh, 1987). The computer softwares, i.e. SPSS, Excel, C+, Minitab and Data Plot are used in this study.

Methodology

The parameters of Gumbel distribution are estimated by MOM, MLM and PWM, whereas the parameters of GEV are estimated by MOM and PWM. Quantile estimates (XT) and their S. E's which correspond to different return periods up to 100 years are also calculated.

In order to test how well the theoretical probability distribution fits the empirical distributions, χ^2 , S-K and PPCC are conducted on the full data. Coefficients of Skewness, Kurtosis and L-moment ratios are also used to justify the fitting of distributions on the data.

Gumbel Distribution

EVI or Gumbel distribution is the most widely used distribution in flood frequency analysis (Rao, 2000). The probability density function $f(x)$ and the distribution function $F(x)$ are:

$$f(x) = \frac{1}{\alpha} \exp\left[-\left(\frac{x-\beta}{\alpha}\right) - e^{-\left(\frac{x-\beta}{\alpha}\right)} \right] \quad \text{... (2.1)}$$

$$F(x) = \exp\left[-e^{-\left(\frac{x-\beta}{\alpha}\right)} \right] \quad \text{... (2.2)}$$

β, the location parameter, is the mode of the distribution. It depends on variance and mean ($\partial f / \partial x = 0$ for $x = \beta$). α is a measure of dispersion and depends only on the variance of x.

$\mu = b + n_e \alpha$ and $\sigma^2 = \frac{\pi^2 \alpha^2}{6}$, where $n_e = 0.5772$

Parameter estimation

MOM METHOD

$$\hat{\alpha} = \frac{\sqrt{6}}{\pi} \sqrt{m_2} = 0.7797 \sqrt{m_2} \quad \text{... (2.3)}$$

$$\hat{\beta} = m_1' - 0.45005 \sqrt{m_2} \quad \text{... (2.4)}$$

Where m_1 and m_2 are moments of 1st and 2nd order.

MLM Method:

$$F(\alpha) = \sum_{i=1}^{N} x_i e^{\left(\frac{-x_i}{\alpha}\right)} - \left(\frac{1}{N} \sum_{i=1}^{N} x_i - \alpha \right) \sum_{i=1}^{N} e^{\left(\frac{-x_i}{\alpha}\right)} = 0 \quad \text{... (2.5)}$$

$$\hat{\beta} \quad \hat{\alpha} \log \left[\frac{N}{\sum_{i=1}^{N} e^{(-x_i/\hat{\alpha})}} \right] \quad \text{... (2.6)}$$

In order to obtain the estimate of parameter α, a computer program in C + + language has been developed and is available with the authors .

PWM methods:

$$\hat{\alpha} = (2b_1 - b_0) / \log(2) = l_2 / \log(2) \quad \text{... (2.7)}$$

$$\hat{\beta} = b_0 - 0.5772157\ \hat{\alpha} \quad \text{... (2.8)}$$

where b_0 and b_1 are plotting estimates and l_2 is sample L-moment.

Quantile Estimates:

The distribution function of EVI (2) can be obtained in the inverse form as

$$x = \beta - \alpha \log(-\log F) \quad \text{... (2.9)}$$

The T- year quantiles, where T is return period, are

$$\hat{x}_T = \hat{\beta} + \hat{\alpha} \log[-\log(1 - 1/T)] \quad \text{... (2.10)}$$

Standard Errors of Quantile Estimates:

For MOM: $S_T^{\,2} = \dfrac{\alpha^2}{N}[1.15894 + 0.19187Y + 1.1Y^2]$ ... (2.11)

For MLM: $S_T^{\,2} = \dfrac{\alpha^2}{N}[1.1087 + 0.5140Y + 0.06079Y^2]$ ... (2.12)

For PWM: $S_T^{\,2} = \dfrac{\alpha^2}{N}[1.1128 + 0.4574Y + 0.8046Y^2]$ ... (2.13)

$$Y = \frac{\partial x}{\partial \alpha} = -\log[-\log(1 - 1/T)]$$

GEV:

$$F(x) = \exp\left\{-\left[1 - k\left(\frac{x - \mu}{\alpha}\right)\right]^{1/k}\right\},\ k \geq 0$$

$$= \exp\left\{-\exp\left[\frac{(x - \mu)}{\alpha}\right]^{k}\right\},\ k < 0 \quad \text{... (2.14)}$$

with x bounded by $\mu + \alpha / k$ from above if $k > 0$ and from below if $k < 0$. Here, μ and α are location and scale parameters, respectively, and k is the shape parameter.

When $k = 0$, the GEV distribution reduces to the Gumbel distribution.
The inverse distribution function is

$$x(F) = \mu + \alpha\{1 - (-\log F)^{k}\} / k, \ k = 0$$

$$= \mu - \alpha \log(-\log F) \qquad \dots (2.15)$$

The moments of the GEV distribution can be expressed in terms of the gamma function, $\Gamma(.)$. For $k>-1/3$, the mean, variance and C_s are given as:

$$\mu_s = \xi + (\alpha / k)\{1 - \Gamma(1 + \xi)\} \qquad \dots (2.16)$$

$$\sigma^2_x = (\alpha / k)^2 \{\Gamma(1+2k) - (\Gamma(1+k))^2\} \qquad \dots (2.17)$$

$$C_s = \frac{Sign(k)\{-1(1+3k) + 3\Gamma(1+k)(1+2k) - 2\Gamma^3(1+k)\}}{\{\Gamma(1+2k) - \Gamma^2(1+k)\}^{3/2}} \qquad \dots (2.18)$$

Parameter Estimation

MOM:

Approximate relationship between the value of k and S_K obtained through Regression Analysis is as under:

For $k > 0$ $(-2 < Cs < 1.14)$, $R^2 = 1$, k is given by

$$K = 0.277648 - 0.322016Cs + 0.060278Cs^2 + 0.016759\,Cs^3$$
$$-0.005873\,Cs^4 - 0.00244\,Cs^5 - 0.00005Cs^6 \qquad \dots (2.19)$$

The initial value k_o is updated by the equations

$$K_{n+1} = k_n - F(k_n) / F^{/}(k_n), \text{Where} \qquad \dots (2.20)$$

$$F(k) = -C_s + \frac{k}{|k|} - \frac{g_3 + 3g_1 g_2 - 2g_1^3}{(g_2 - g_1^2)^{3/2}} \qquad \dots (2.21)$$

$$F(k) = \frac{k}{|k|} \; \frac{-3d_3 + 3d_1 g_2 + 6g_1 d_2 - 6g_1^2 d_1 - 3k(-g_3 + 3g_1 g_2 - 2g_1^3)(2d_2 - 2g_1 d_1)}{(g_2 - g_1^2)^{3/2} \; 2|k| (g_2 - g_1^2)^{5/2}} \qquad \dots (2.22)$$

$$g_r = \Gamma(1 + r k), \text{ and} \qquad \dots (2.23)$$

$$d_r = \Gamma^{/}(1 + r k) = \Gamma(1 + r k)\,\psi(1 + r k) \qquad \dots (2.24)$$

The iteration in equation (2.20) is repeated until $F(k)$ is sufficiently close to zero. Once k is estimated we get $\hat{\alpha}$ and $\hat{\mu}$ as

$$\hat{\alpha} = \left[m_2 \hat{k}^2 / \{ \Gamma(1+2\hat{k}) - \Gamma^2(1+\hat{k}) \} \right]^{1/2} \quad \text{... (2.25)}$$

$$\hat{\mu} = m'_1 . \frac{\hat{\alpha}}{\hat{k}} \left[1 - \Gamma(1+\hat{k}) \right] \quad \text{... (2.26)}$$

PWM:

$$\beta_r = (r+1)^{-1} \left[\mu + \frac{\alpha}{k} \{ 1 - (r+1)^{-k} \Gamma(1+k) \} \right] \quad \text{... (2.27)}$$

$$\hat{k} = 7.859C + 2.9554C^2, \text{ (Approximate value)} \quad \text{... (2.28)}$$

$$C = \frac{2b_1 - b_0}{3b_2 - b_0} - \frac{\log^2}{\log^3} \quad \text{...(2.29)}$$

(b_0, b_1, b_2 are the sample estimates of β_o . β_1. β_2.)

$$\hat{\alpha} = \frac{(2b_1 - b_0)\ \hat{k}}{\hat{\Gamma}\ (1+\hat{k})(1-2^{-\hat{k}})} \quad \text{...(2.30)}$$

$$\hat{\xi} = b_0 + \hat{\alpha} \{ \Gamma(1+\hat{k}) - 1 \} / \hat{k} \quad \text{... (2.31)}$$

Quantile Estimates:

$$x\ \xi + \frac{\alpha}{k} \left[1 - (-\log F)^k \right] \quad \text{... (2.32)}$$

$$\hat{x}_T = \hat{\xi} + \frac{\hat{\alpha}}{\hat{k}} \left[1 - \left\{ -\log\left(1 - \frac{1}{T} \right) \right\} \right] \quad \text{... (2.33)}$$

Note : The formulae (2.3.2) to (2.33) for parameter estimation, Quantile estimation and their S . E's used in this paper are fully explained and derived (Rao,2000). The results are obtained by using Excel, C+ + language and Minitab softwares. The S. E's for GEV are not calculated.

Goodness of fit tests:

The details for conducting χ^2 – test, S-K test and PPCC are discussed earlier (Rao, 2000; Memon and Shaikh 2004, a).
The results are obtained using Data Plot Software and are provided in Table-IV.

Role of Skewness, Kurtosis and L-moment Ratios:

C_s and C_k

(Rao 2000) suggested the relationship between C_k and C_s for identification of parent probability distributions as follows:

Cs:	0.0	2.0	1.14	0.0	0.0
Ck:	1.8	9.0	5.40	4.2	3.0
Distribution:	Uniform	Exponential	Gumbel	Logistic	Normal

Also for GEV distribution the relationship is (approximately):

$$C_k = 2.6951 + 0.185C_s + 1.7534C_s^2 + 0.1107C_s^3 + 0.0377C_s^4 + 0.0036C_s^5 + 0.0022C_s^6 \quad \text{... (2.34)}$$

L-Moment Ratios

Hosking (1997) has developed relationship between L-moment ratio t_3 and t_4, for approximating probability distributions as follows:

t_3:	0.0	0.333	0.17	0.00	0.00
t_4:	0.0	0.167	0.15	0.167	0.123
Distribution:	Uniform	Exponential	Gumbel	Logistic	Normal

Also for GEV distribution the relationship is (approximately):

$$t_3 = 0.107 + 0.1109t_3 + 0.8484\, t_3^{\,2} - 0.667\, t_3^3 + 0.0057\, t_3^4 - 0.0421t_3^5 + \quad \text{... (2.35)}$$

These relationships give some idea about candidate probability distribution to be fitted on any given data.

Results and Discussion

Descriptive Statistics:

Fig.1 shows that super floods of 1.1 million cusecs or more have occurred in the years 1976, 1986 and 1988. Fig. 2 shows that high floods between the ranges of 870-980, and 1090-1200 thousand cusecs have occurred 1,4 and 4 times, respectively, from 1962 to 1999. Various statistics presented in Table:1 reveal that there is a substantial difference between mean and median, which increases with increase in sample size. This is in confirmation to the results given by Nixon. (Bund Manual, 1977).The coefficient of Skewness (Cs)is 0.612 for N= 24 which indicates that the distribution is + vely skewed.

Parameter and Quantile Estimates (Table:2 and 3):

The estimated parameters of Gumbel distribution by MOM, MLM and GEV distribution by MOM and PWM are presented in Table: 2.

Table: 3 shows that Quantile estimates using Gumbel by PWN yield higher floods than by MLM and MOM, in order. PWN estimates yield slightly higher floods than by MOM. The differences between them vary from 0.1% to 2%, less for GEV than for Gumbel. Quantile estimates for GEV > until estimates for Gumbel by each method. (the differences being very small).

1.50 million cusecs flood is expected at Guddu during next 100 years, by Gumbel, using MLM; and 1.515 million cusecs flood is expected by GEV using PWM, which is 0.4 million cusecs (38%) above the designed capacity. It should be noted that Gumbel's extreme flood frequency curve using data for 24 years estimated 1.4 million cusec floods for 100 years (Bhutto and Shaikh, 1987).

S. E's of Quantile Estimates by Gumbel (Table: 3)

For T = 5, 10, 20, 25, 50, 75 and 100 the ratio are:

$S.E_{(MOM)} / S.E_{(MLM)} =$ 1.08, 1.14, 1.17 1.18, 1.20, 1.21 and 1.22

$S.E_{(PWM)} / S.E_{(MLM)} =$1.07, 1.10, 1.12 1. 12, 1.13, 1.13 and 1.14

Hence MLM is "the best" and MOM is 'the worst'. This is in confirmation to the theory as the sample size is medium (Rao 2000).

Goodness of Fit tests (Table: 4)

Gumbel distribution is suitable on the basis of χ^2 – test at 5% level and also on the basis of PPCC (being approximate equal to 1.0). It is rejected on the basis of S-K test, but as S-K test is not a powerfull test (Rao,2000), so Gumbel distribution seems to be a proper distribution.

GEV distribution is not appropriate as it is rejected by χ^2-test, S-K test and PPCC. Thus we have not calculated S.E's of Quantiles by GEV.

Role of C_s, C_k and L -Moment Ratios

Table: 5 Observed and suggested values of C_s, C_k, t_3 and t_4 (Guddu)

	Observed	For Gumbel (Rao, 2000)	Appx.Value For GEV
t_3	0.153	0.173	0.153
t_4	0.116	0.150	0.436*
C_s	0.612	1.14*	0.612
C_k	2.326	5.40*	3.492*

* indicates rejection of Null Hypothesis (the difference between observed and suggested values becomes greater than 20%)

Thus Gumbel is not proper distribution on the basis of C_s and C_k, while GEV is not a proper fit on the basis of t4 and C_k

Note: Idrees (1994) had neither used S. E's nor χ^2-test and PPCC test (which is most reliable). Moreover he used only PWM method for estimation and S-K test as Goodness of fit test. Hence, their suggestion for using GEV is not acceptable.

Conclusion

1. PWM quantile estimates are higher than MOM quantile estimates, for both distributions. Quantile estimates for GEV are slightly higher than for Gumbel distributions.

2. MLM is the best and MOM is the worst on the basis of S.E's of Quantiles, by Gumbel.

3. GEV is not appropriate (χ^2-test and PPCC test). Gumbel is a proper fit (χ^2-test, PPCC t_4 and C_k,t_3 and t_4).

4. 1.50 million cusecs floods is expected during 100 years by Gumbel distribution, using MLM method of estimation, as compared to its designed capacity of 1.1 million cusecs. Hence, there is a need for remodeling the Guddu barrage or taking suitable steps to save the structure of barrage.

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Table 1: Basic Statistics (Guddu barrage, 1962 – 1999).

38 (24)	Mean*	Median*	Mode*	Min*	Max*
	701.7(665.0)	651.5 (630)	613.0(613.0)	324.0(349.8)	1200.0(1200)
Cs	C_k	S. D	Q. D	C.V	S.E
0.612(0.523)	2.326(1.916)	251.3(232.7)	114.0(114.0)	0.36(0.26)	42.9 (39.3)
Q_1*	Q_2*	D_1*	D_9 *	t_3	t_4
537.0(503.0)	825.0(731.0)	405.0(369.0)	1157.0(1120.0)	0.153	0.116

Note: The values in the parenthesis are for 24 years.

* indicates values in (000 Cusecs).

Table 2: Parameters of Gumbel and GEV Distributions (in 000 cusecs) at Guddu (1962–1999)

Distribution	Parameter	MOM	MLM	PWM
Gumbel	$\hat{\alpha}$	195.776	199.515	203.782
	$\hat{\beta}$	588.608	585.755	584.032
GEV	$\hat{\mu}$	618.52	……..	615.678
	$\hat{\alpha}$	206.56	……..	208.0164
	$\hat{K}$	0.028	……..	0.027

Table 3: Quantile estimates and their S.E's (in parentheses) by Gumbel and GEV distributions at (Guddu barrage, 1962 – 1999).

Return Period T	Exceedence Probability	Quantile Magnitudes (in 000 cusees)				
		Gumbel			GEV	
		MOM	MLM	PWM	MOM	PWM
5	.20	882.26 (62.89)s	885.01 (58.32)	889.69 (62.80)	920.4	921.5
10	.10	1029.17 (84.99)	1034.74 (74.82)	1042.61 (82.42)	1066.8	1069.8
20	.05	1170.10 (107.38)	1178.35 (91.53)	1189.30 (102.26)	1202.1	1209.4
25	.04	1214.80 (114.63)	1223.91 (96.94)	1235.83 (108.67)	1244.6	1253.1
50	.02	1352.51 (137.17)	1364.25 (113.83)	1379.17 (128.66)	1374.8	1386.0
75	.013	1432.56 (150.40)	1445.82 (123.75)	1462.49 (140.39)	1448.3	1462.2
100	.01	1489.21 (159.81)	1503.55 (130.82)	1521.4 (148.73)	1500.1	1515.0

Table 4: Tests of Goodness of Fit (Guddu Barrage, 1962 – 1999).

χ^2-test		S – K Test		PPCC
α=5%	χ^2 cal Gumbel GEV	α= 5%	Dcal Gumbel GEV	Gumbel GEV
5.99	4.083 10.82*	.21	.290* .280*	.974 .682*

* indicates rejection of Null Hypothesis.

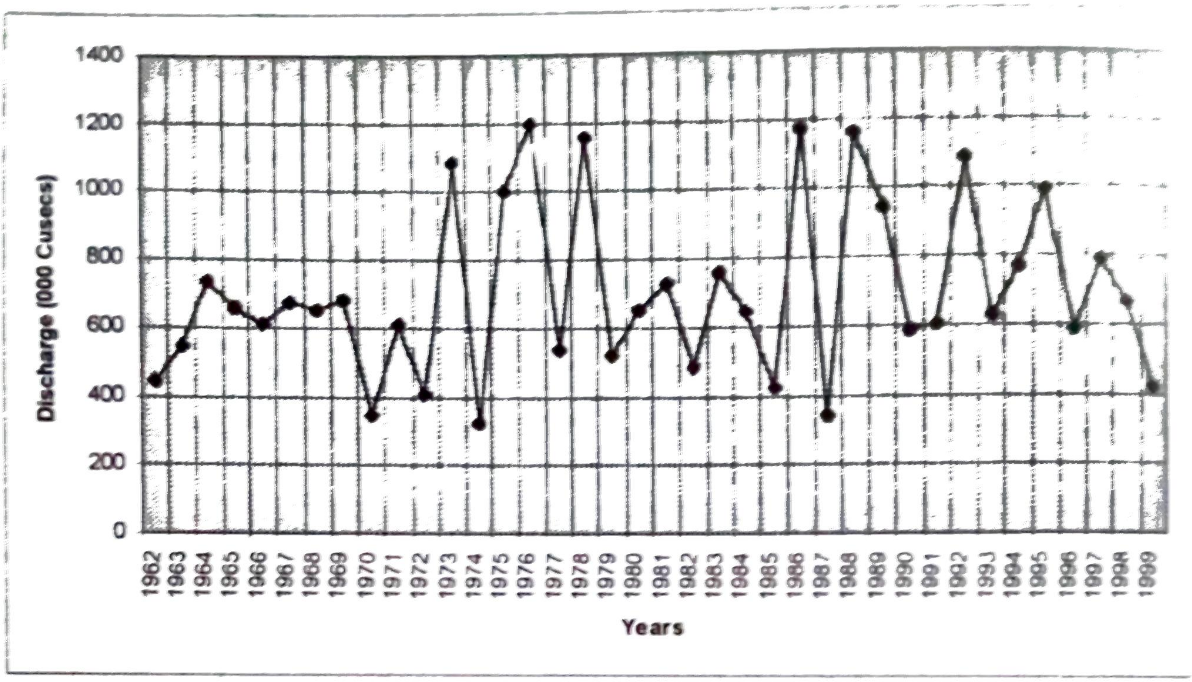

Fig.1 Peak floods discharge at Guddu barrage (1962-1999)

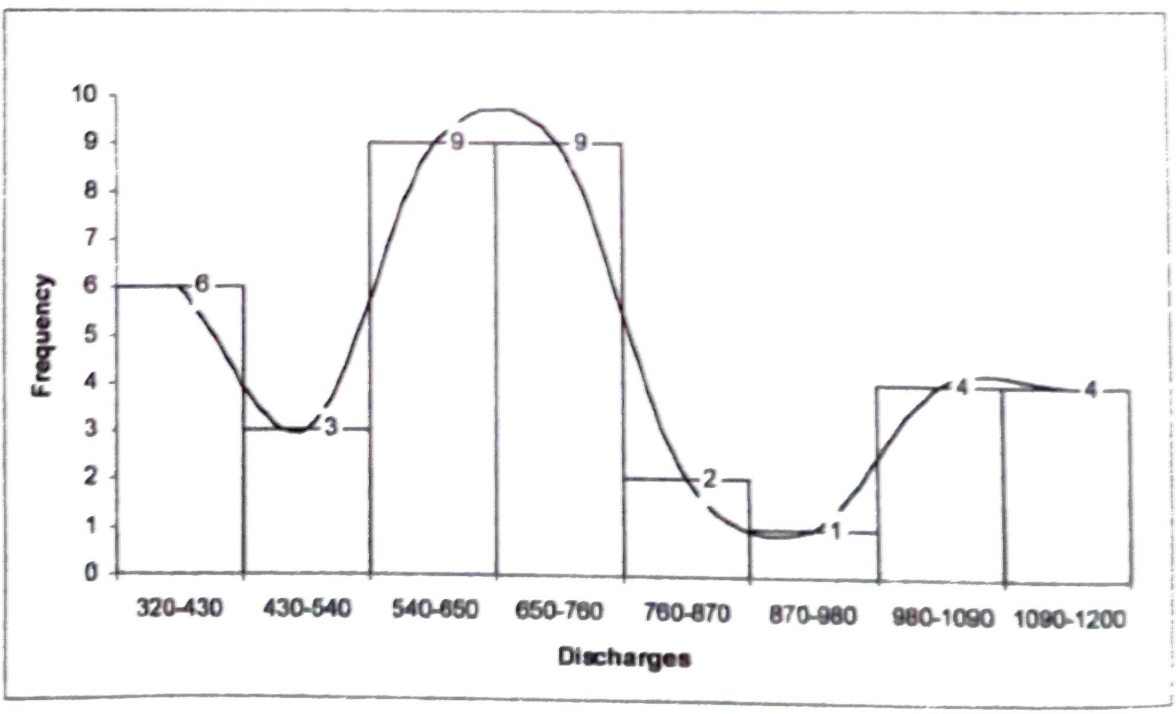

Fig.2 Frequency curve of flood peaks at Guddu barrage (1962-1999)

References

Abul Sabur, M. (1982) Regional Flood Frequency Analysis of Thailan, M. E. Thesis, AIT, Bangkok.

Ahmed, M. I., C.D. Sinchair and A. Werrity, (1988) Log-Logistic Flood Frequency Analysis . J.Hydrol. **98:** 205-224.

Bhutto, H. B. and N. M., Shaikh, (1987) Flood Frequency Analysis of the Indus River at Guddu Barrage. Mehran Univ. Res. Jour. Eng. Technol. **6:** (2) 25-33.

Bund Manual, (1977) Irrigation and Power department, Government of Sindh.

Chowdhury, J. (1995) Flood Frequency Analysis. Institute of Flood Control and Drainage Research, Bangladesh University of Engineering and Technology.

Chowdhury and M. Abdul Karim, (1995) A Comparison of Four Distributions used in Flood Frequency Analysis in Bangladesh. Hyrlog. Sci. J. **40:** 55-65

Cunnane,C. (1989) Statistical Distributions For Flood Frequency Analysis. World Meteorological Organization, WMO-No: 718, Hydrology Report No:03, Geneva, Switzerland.

Fatih, T. G. (200) Determining Suitable Probability distribution Models for Flow and Precipitation Series of the Seyhan River Basin.Turk. Agricult: 187-194.

Haktanir,T.(1991) Statistical Modeling of Maximum Flows in Turkish River Hrrol. Sci. J. **36** (4): 367-389.

Hosking, J. R. M. (1997) Regional Frequency Analysis: An Approach Based on L-Moments, Cambridge University Press.

Hosking, J. R. M; J. R . Walis and E. F. Wood, (1985) Estimation of the Generalized Extreme-Value Distribution by the Method of Probability-Weighted Moments. Technometrics, **27:** 251 - 61.

Idrees, M.(1994) Frequency Prediction Analysis of Flood Data of Jhelum River at Rasul by Genaralized Extreme Value Distributions Pak. J. Agricult. Sci. **31**, (3): 33-45.

Jain, D, and V.P. Singh, (1986) A Comparison of Transformation Methods for Flood Frequency Analysis. Water Resources Bulletin,**22** (6): 903-912.

Jenkinson, A. F. (1955). The Frequency Distribution of the Annual Maximum (or Minimum) Value of Meterological Elements. Quat J. Royal Metrolog. Soc. **87**: 158pp.

Landwehr, J. M; N. C. Matalase, and J. R. Wallis, (1979) Probability Weighted Moments Compared with some Traditional technique in estimating Gumbel Parameters and Quantiles. Water Resources Research,**15** (5): 1055-1064.

Lowery, M. and J. E. Nash, (1970) A Comparison of Methods of Fitting the double Exponential Distribution. J. Hydraul. **10:** 259-275.

Memon, A. G. and N. M. Shaikh, (2005) Frequency Prediction Analysis of Flood Data of Indus River At Kotri Barrage by Gumbel and Genaralized Extreme Value Distributions. Mehran Univ. Res. J. Engin .and Technol. **24** (1): 23-35.

Memon, A. G. and N. M. Shaikh, (2005) Fitting Gumbel and GEV(2) Probability Distributions on Floods Data of Indus River At Sukkur Barrage. J. Pure and Appl. Sci. Islamia University, Bahawalpur, **3:** 1-11.

Onoz, B. an M. Baryazit, (1995) Best-fit distributions of Largest Available Flood Samples J. Hyrol. **167:** 195-208.

Rao, R. A. (2000) Flood Frequency Analysis. CRC Press, London.

Yang Cheng-Te, (1987) Flood Prediction System in a River Basin with Tidal Influence. M. E. Thesis, AIT, Bangkok.