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COMPUTER SIMULATION FOR 3–D MATHEMATICAL MODEL USED IN ROBOTIC END-EFFECTOR

Zulfiqar Ali Soomro

Mechanical Engineering Department Quaid-e-Awam, University of Engineering Sciences & Technology Nawabshah

Abstract

This paper evolves the running of computer program using Matlab software based upon the simulated values, calculated for 3-D mathematical model used in robotic End–Effector. The model consists of six legs, attached by prismatic joints in middle and by spherical joints with two circular platforms. This model has capacity to produce mobility three in line directions and three in concerned orientations. Here line vectors obtained by cross product of unit vectors (conversion of local coordinates into global coordinates) in shape of columns make Jacobian matrix. The unknown force is calculated by stiffness and concerned displacements of legs. These all factors involved to calculate wrench analysis. Thus these all complicated terms were solved in matrix form by simulating values through Matlab program.

Keywords: Jacobin Matrix, Congruence Matrix, wrench analysis

Introduction

According to Cartesian coordinate system, which is applied upon the proposed 3–D robotic model, it has also capability to move in X, Y, Z directions.

In this project, it has determined the coordinate system at elevation as well as at top of the model. it also determined the local coordinates of any leg as reference for global co-ordinate system. Thus local coordinate system is converted into global coordinate system [1].

These determined stiffnesses (K) were used for practical and theoretical analysis. For practically stiffness mapping, it gave required local displacements depending on its applied loads [2].

The Kinematics analysis is used to determine unit vector (V) and position vector (r) of each leg. The line vectors which are consisting upon the cross product of unit vectors and position vectors along with unit vectors to give six elements of each leg for Jacobian matrix. Thus Jacobian matrix (J) is determined consisting of 6x6 matrix used for determination of displacements [3].

These local and global displacements are co-related and calculated by using wrench analysis (δw) in above sections. However initially load was applied vertically and centrally downward parallel to z-axis. Thus local displacements (δd_L) were determined Experimentally. Then global displacements (δD_g) are calculated by using the relationship through wrench analysis. Thus theoretical results of local displacements from global displacement by congruence (K_g) matrix of wrench analysis are achieved [4].

The factors like local displacements and global displacements used for making unit vectors. These unit vectors (V), stiffnesses (K), Jacobian matrix (J) and wrench analysis (δw) are used for modification of system in shape of the computer simulation in subsequent sections [5, 6].

2. Methodology

The criteria for formulating the computer simulation by 'Matlab' depends upon the

mathematical phenomenon for robotic model. This mathematical model comprises of stiffness matrix, Jacobian matrix and wrench analysis will be discussed in subsequent section other place [6]. Here computer simulation of these factors is demonstrated by 'Mat lab' software as under.

This computer program starts from line vectors (V1 to V6) depending upon conversion of local coordinates to global coordinates by rotation matrix through concerned trigonometric identities.

v1 = [cos (pi/3)* cos (pi/6) cos (pi/3)* sin (pi/6) sin (pi/3)]

v2 = [cos (pi/3)* cos (pi/3)* cos (pi/6) - sin (pi/3)* cos (pi/3)* sin (pi/6)

sin (pi/3)* cos (pi/ 3)* cos (pi/6) + cos (pi /3)* cos (pi/3)* sin (pi/6) sin pi/3)]

v3 = [-sin (pi/3)* cos (pi/3)* cos (pi/6) – cos (pi/3)* cos (pi/3)* sin (pi/6)

cos (pi/3)* cos (pi/3)* cos(pi/6) - sin (pi/3)* cos(pi/3)* sin (pi/6) sin (pi/3)]

v4 = [-cos (pi/3)* cos(pi/6) – cos(pi/3)* sin(pi/6) sin(pi/3)]

v5 = [-cos (pi/3)* cos (pi/3)* cos(pi/6) + sin(pi/3)* cos(pi/3)* sin(pi/6)

sin (pi/3)* cospi/3)* cos-(pi/6) – cos (pi/3)* cos(pi/3)* sin (pi/6) sin (pi/3)]

v6 = [sin(pi/3)* cos(pi/3)* cos(pi/6) + cos(pi/3)* cos(3)* sin(pi/6)

cos(pi/3)* cos(pi/3)* cos(pi/6) + sin(pi/3)*cos (pi3)*sin (pi/6) sin (pi/3)]

Then position vectors (r_1 to r_6) are obtained from design of 3–D model, keeping the constant distance from each local coordinate with respect to central reference i-e 69.87mm by trigonometric analysis.

r1 = [-69.87*cos (pi / 6) – 69.87* sin (pi / 6) 0]

r2 = [0 - 69.87 0]

r3 = [69.87*cos(pi / 6) – 69.87* sin(pi / 6) 0]

r4 = 69.87*cos(pi / 6) 69.87*sin(pi / 6) 0]

r 6 [-.69.87*cos(pi / 6) 69.87* sin (pi / 6) 0]

By means of cross product of unit vectors (V1 to V6) and position vectors (r1 to r6) lin vectors (1v1 to 1v6) are formed as under

1v1 = cross (r 2, v 1)

1v2 = cross (r 2, v 2)

1v3 = cross (r 3, v 3)

1v4 = cross (r 4, v 4)

1v5 = cross (r 5, v 5)

1v6 = cross (r 6, v 6)

Thus these line vectors help to form Jacobian matrix (J) and its transpose (J_t) as under:

J_t = [v1,1v1; v2,1v2; v3, 1v3; v4, 1v4; v5,1v5; v6, 1v6] J = transpose (J_t)

In following sections the elements of the stiffness matrix (K) are shown. Since this matrix is diagonal matrix and its off-diagonal elements are zero.

K = [9.81*0.1437060 0 0 0 0; 09.81* 0.149613 0 0 0 0;0 0 9.81* 0.14613 0 0 0 ; 0 0 0 9.81*0.36719 0 0; 0 0 0 0 9.81* 0.135348 0;0 0 0 0 0 9.81* 0.135859]

In above matrix each diagonal element is multiplied to convert units in S.I system in below wrench the value 9.81 N denotes load in Z–direction, when it acts centrally in vertical way only, while-shows upwards direction.

W = [0., 0.,-9.81 ; 0; 0; 0]

The congruence matrix (K_g) is the stiffness with Jacobian matrix and its transpose as under.

$K_g = J^* k^* Jt$

The inverse of the congruence matrix is taken for determining the global displacement (L) with wrench (w) and local displacement (ℓ) by multiplication of Jacobian transpose (J_t) with global displacement (L).

h = inv (kg)

L = h* w

I = Jt* L

Thus force (f) can be determined easily for all six legs by multiplying stiffness with local displacement.

f = k*l

If any want to find un-known force of any leg, one can do that easily. An example is given as under. Here if applied un-known force on one leg individually which covers distance (d) and thus one calculate un-known applied force on leg.

d = [2.75; 0; 0; 0; 0; 0]

f1= k*d

This calculated force (F) is utilized to determine the total wrench by multiplying it with jacobian matrix as under.

F = [0.3951915;0;0;0;0;0]

W =J*F

Thus these simulated values by Matlab' can help to run program on computer for determining and calculating very complicated problems of jacobian, congruence and wrench analysis in matrix form.

Results

This computer simulation is based upon the mathematical model parameters denoted in fig. 2 through trigonometric identities sketched from fig.1. thus unit vectors (v1 to v6) and position vectors (r1 to r6) were calculated to give line vectors (1v1 to 1v6) for Jacobian matrix, its transpose, and wrench. By simulating these values by 'Matlab' on computer, it can run program and can give required results as mentioned in Appendix.

The relation between global and local displacements was computed by 'Matlab' is also simulated in Table-1 and displayed in Graph-1. Besides this, relationship between force (f) and wrench (w) is enumerated in Table-1 dependant upon the result of computer simulation. This relationship between these forces and wrench is traced in graph-2.

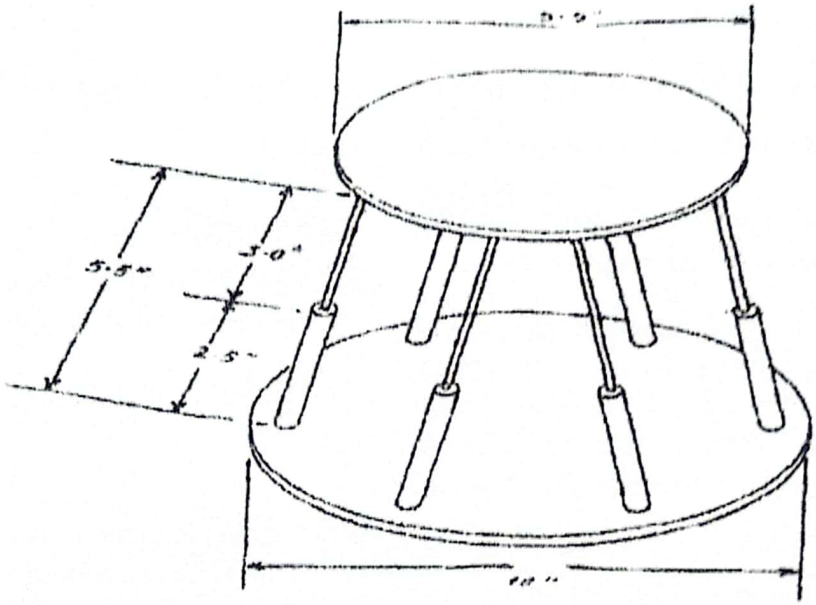


Fig.1 Robotic 3-D Mathematical Model used in robotic End-Effector

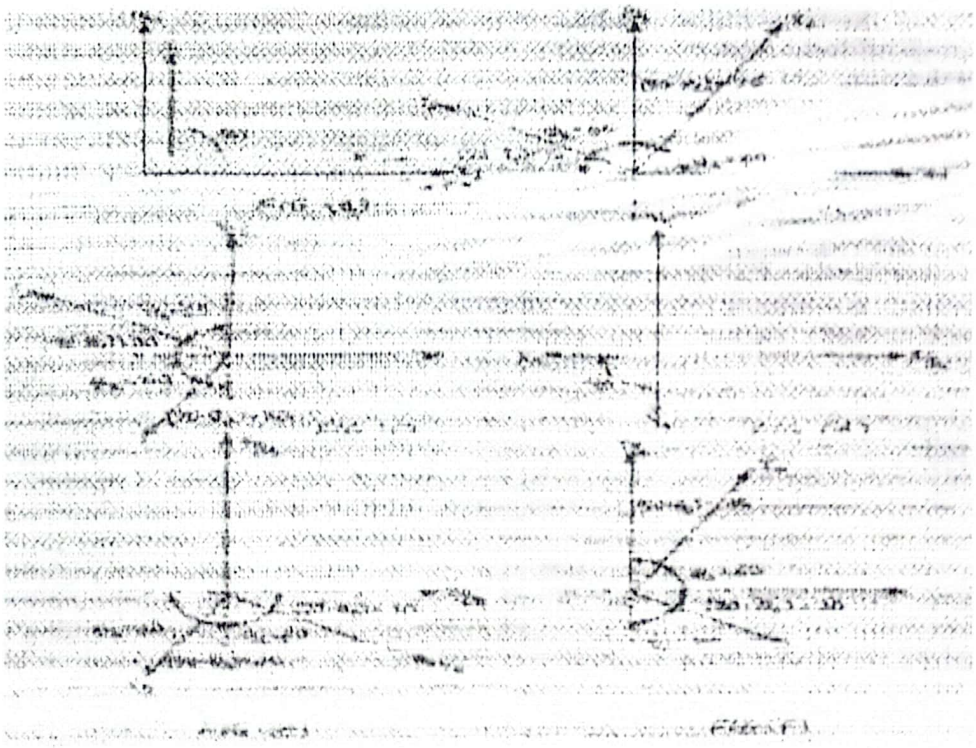
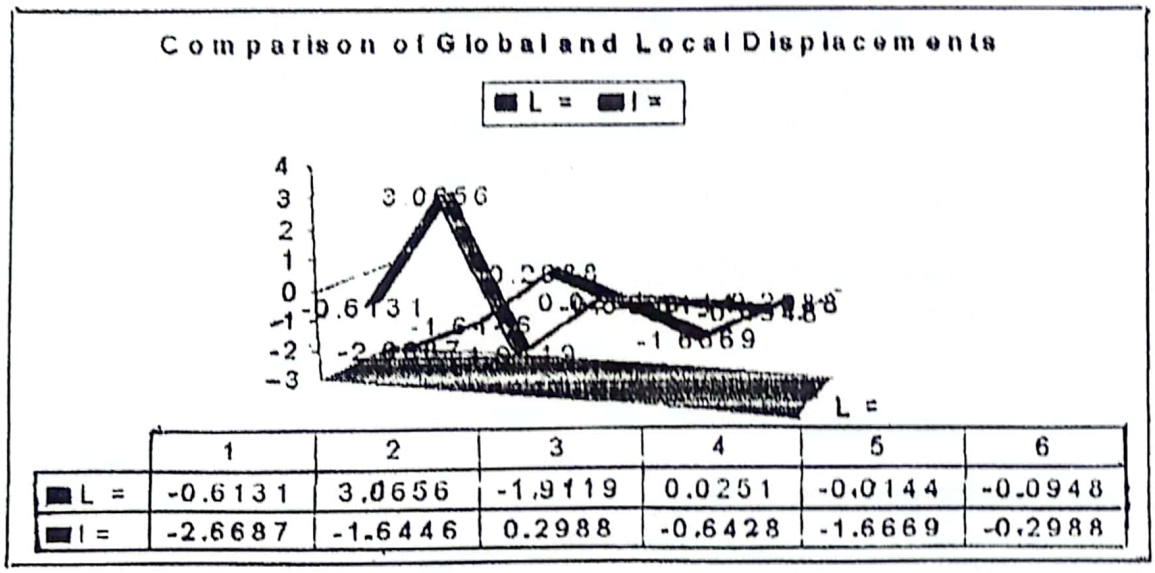


Fig.2 Trigonometric relation of local and global domain for each leg of model

Global displacement (L)	Local Displacement (1)	Determination Force (f)	Calculated Wrench (w)
-0.6131	-2.6687	-3.7622	0.1711
3.06131	-1.6446	-2.4138	0.0988
-1.9119	0.2988	0.4283	0.3422
0.0251	-0.6428	-2.3155	-11.9564
-0.0144	-1.6669	-2.2133	20.7090
-0.0948	-0.2988	-0.3982	0

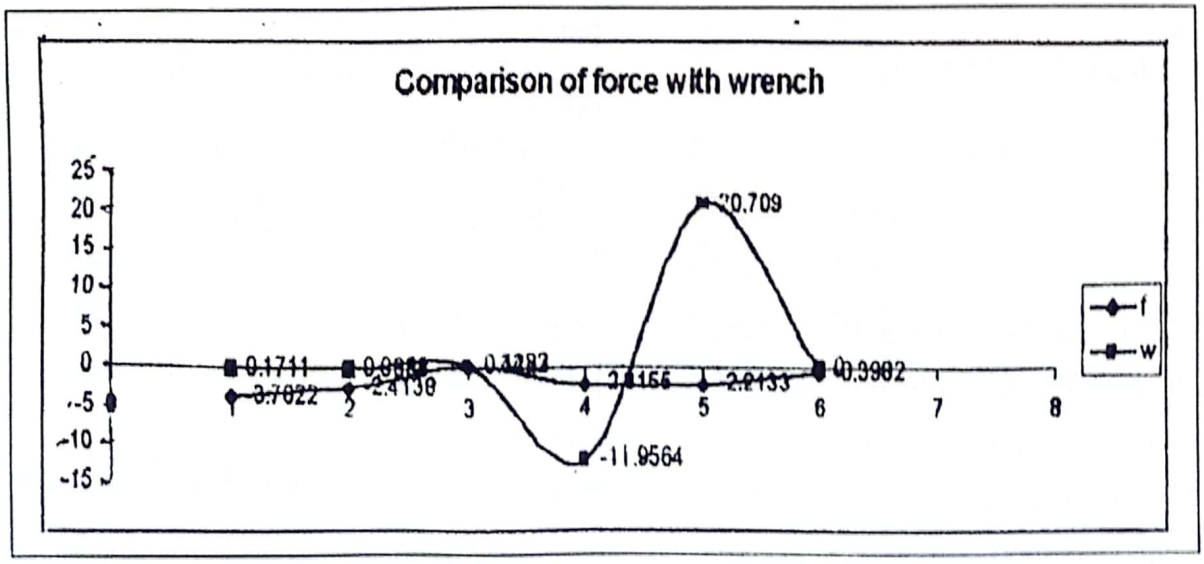
Table –1 Calculated Parameters from Computer Programming

4. Here higher value of global displacement is 3.0656 and lower value is – 0.0144 while 0.2988 is higher and –2.6687 as lower value for local displacement. Similarly 0.4283 and 20.7090 are higher while –3.7622 and –11.9564 are lower values for force and wrench respectively.



	1	2	3	4	5	6
L =	-0.6131	3.0656	-1.9119	0.0251	-0.0144	-0.0948
I =	-2.6687	-1.6446	0.2988	-0.6428	-1.6669	-0.2988

Graph-1 showing relation between Global and Local displacements



Graph-2 Relation between Outputs of Force and Wrench analysis

Discussion

The 3–D model, which is used in robotic end-Effector, has six legs attached with themselves by prismatic joints and fitted with two platforms by spherical joints as shown in fig-1. These individual legs treat as local coordinates moving in three translatory and rotatory directions in X, Y, Z planes. These local coordinates are transformed into global coordinates to give unit vectors as shown in fig.2. The position vectors are calculated to make cross product to give jacobian matrix. The stiffnesses for springs inserted inside the legs were experimentally observed in laboratory. The stiffness in the form of matrix can calculate the un-known forces by their concerned local displacements. Thus by means of jacobian and congruence matrix wrench analysis are determined. Using 'Matlab' software solves this whole phenomenon. These above simulated values if feeded properly, can run the concerned program on computer as shown in Appendix. These computer simulated values are used to solve complicated values in form of 6x6 matrix of multiplication.

APPENDIX (Computer Simulation in 'MATLAB' program).

»v1 =
0.4330 0.2500 0.8660
v2 =
0.0000 0.5000 0.8660
v3 =
-0.5000 0,0000 0.8660
v4 =
-0.4330 -0.2500 0,8660
v5 =
-0.0000 -0.5000 0.8660
v6 =
0.5000 -0.0000 0.8660
r1 =
-60.5092 -34.9350 0
r2 =
0. -69.8700 0
r3 =
60.5092 –34.9350 0
r4 =
60.5092 34.9350 0
r5 =
0 69.8700 0
r6 =
-60.5092 34.9350 0
1v1 =
-30.2546 52.4025 0
1v2 =
-60.5092 0 0.0000
1v3 =
-30.2546 -52.4025 -17.4675
1v4 =
30.2546 -52.4025 0
1v5 =
60.5092 0 0.0000
1v6 =
30.2546 52.4025 -17.4675

Jt =

0.4330	0.2500	0.8660	-30.2546	52.4025	0
0.0000	0.5000	0.8660	- 60.5092	0	0.00
- 0.5000	0.0000	0.8660	30 2546	-52.4025	-17.4675
-0.4330	-0.2500	0.8660	30.2546	-52.4025	0
-0.0000	0.5000	0.8660	60. 50.92	0	0.00
0.5000	0.0000	0.8660	30.2546	52. 40.25	-17.4675

J =

0.4330	0.0000	-0.5000	-0.43330	-0.0000	0.5000
0.2500	0.5000	0.0000	-0.2500	-0.5000	-0.0000
0.8660	0.8660	0.8660	0.8660	0.8660	0.86660
-30.254	-60.509	-30.2546	-30.2546	60.5092	30.2546
52.4025	0	-52.4025	-52.4025	0	52.4025
0	0.00	-17.4675	0	0.00	-17.4675

k=

1.4098	0	0	0	0	0
0	1.4677	0	0	0	0
0	1.4335	0	0	0	0
0	0	0	3.6021	0	0
0	0	0	0	1.3278	0
0	0	0	0	0	1.3328

w =

0
0
-9.8100
0
0
0

kg =

1.0e+004*

0.0002	0.0001	-0.0001	-0.00024	0.0186	0.0001
0.0001	0.0001	-0.0000	-0.0122	0.0066	0.0000
-0.0001	-0.0000	0.0008	0.0047	-0.0104	-0.0042
-0.0024	-0.0122	0.0047	1.7355	-0.3560	0.0053
0.0186	0.0066	-0.0104	-0.3560	2.1359	0.0092
0.0001	0.0000	-0.0042	0.0053	0.0092	0.0644

Warning: Matrix is close to singular or badly scaled

Results may be inaccurate. RCOND = 1.041533e – 020.
In C:\Program Files \Norton Web Services\UndoDir122\C\MATLAB\bin\1ps.m at line 29

h =

1.0e + 015*

-3.5311	0.9462	0.0000	0.0078	0.0292	0.0000
0.9462	-0.2535	-0.0000	-0.0021	-0.0078	-0.0000
0.0000	-0.0000	0.0000	-0.0000	0.0000	0.0000
0.0078	-0.0021	-0.0000	-0.0000	-0.0001	-0.0000
0.0292	-0.0078	0.0000	-0.0001	-0.0002	0.0000
0.0000	-0.0000	0.0000	-0.0000	-0.0000	0.0000

L =

-0.6131
3.0656
-19119
0.0251
-0.0144
-0.0948

f 1 =

3.8768
0
0
0
0
0

I=

-2.6687
-1.6446
0.2988
-0.6428
-1.6669
-0.2988

f =

0.3952
0
0
0
0
0

f =

-3.7622
-2.4138
0.4283
-2.3155
-2.2133
-0.3982

W =

0.1711
0.0988
0.3422
-11.9564
20.7090
0

d =

2.7500
0
0
0
0
0

Conclusion

Simulating the values of unit vectors, position vectors for Jacobian and congruence matrix, makes the 'Matlab' programme. While unknown concerned force for each leg was determined by concerned displacement and stiffness matrix. Finally total wrench analysis was calculated depending upon all these factors. Thus computer simulation on 'Matlab' helped in solving complicated problems in matrix form. Thus a frame formulated in shape of Computer 'Matlab' Program for calculating design of any other type of model. Only simulating the values of variable parameter in specified computer program, a different structural mathematical model can be determined.
The aim of various denotions for same quantity is same as under during calculation due to theory and programme difference detail computer simulated values are manifest in Matrix form, given in Appendix.

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