Sindh Univ. Res. Jour. (Sci.Ser.), Vol: 35(1) 37-48 (2003)

SOME PERFORMANCE OF A REPAIRABLE CIMS TRANSFER LINE WITH TWO MACHINES

Mir G. H. Talpur, Javed, I. Chandio* and Imdad, H. Soomro*

Department of Statistics, Shah Abdul Latif University, Khairpur

Abstract

Consider a repairable CIMS (Computer Integrated Manufacturing System) transfer line consisting of two un reliable machines and a single storage buffer. A new model is taken where two general distributions are discussed. Having deliberated on the entire processing time of particular CIMS transfer line it is found that the model discussed in this paper is equivalent to a queuing system C_2/ G / 1 / N from the view point of the performance measures.

Keywords: CIMS transfer line consisting of two un reliable machines

Introduction

A repairable CIMS transfer line consists of a series of machines separated by buffers. Parts (or work pieces) flow from outside the system to the first machine, than to the first buffer, then to the second machine, then to the second buffer, and so forth until the last machine, after which they exist the system. Each part is required to be processed on each machine during a random amount of time called processing time. If a machine fails when a part is being processed on the machine, the part must wait for counting processing till the failed machine is repaired. The processing time, lifetime and reaper times of any machine are all random variables. This CIMS transfer line has been discussed by many authors, such as references [1], [2], [3], [4] etc. In those papers, the authors obtained the various approximating results when the random variables dealt with are all exponentially distributed.

In this paper, we consider a repairable CIMS transfer line consisting of two unreliable machines and a single storage buffer. There are many papers dealing with this model Gershwin & Berman and Zhang reviewed many works of this model in references [1] and [5] , respectively Here, we discus a new model where two general distributions are considered. By considering the complete processing time of a special CIMS transfer line; we find that the model discussed in the section is equivalent to a queuing system, denoted by C_2 / G / 1 / N, from the viewpoint of performance measures, such as production rate, blocked probability and starved probability. In Section 3, we first analyze the exact solutions of C_2 / G / 1 / N when $N = 0, \infty$, and $1<N< \infty$, respectively. In section 4, we connect the queuing system C_2 / G / 1 / N with the CIMS transfer line, and obtain the explicit results of production rate, blocked probability and starved probability. We find that the results are no relevant to general distributions; in fact, they only depend on the expectations of corresponding random variables. When $N = \infty$ and when $1 < N < \infty$, the results are closely related to the general distributions.

2. Assumptions of CIMS Transfer Line

In this section, we describe the CIMS transfer line discussed as following

(a). The system consists of two machines that are separated by a storage buffer. Work pieces (or parts) enter machine 1 from the outside. Each work pieces is operated upon in machine 1, then passes to the buffer and then processed to machine 2. After being

* Department of Statistics, University of Sindh, Jamshoro

operated on machine 2, the work piece leaves the system.

(b). The processing time, lifetime and repair time of machine I have constant rates μ_1, α_2 and β_1, respectively. The processing time and repair time of machine 2 have general density functions f (*) and r (*) with expectations $1/\ \mu_2$ and $1/\ \beta_2$, respectively. And the lifetime of machine 2 has constant rate α_2.

(c). The amount of material in storage is represented by the integer n, $0< n< N+1$. This is the number of workpieces in the buffer plus the number (that is 0 or 1) of workpieces in machine 2. There are no workpieces in machine 2 if the buffer becomes empty and machine 2 completes the last remaining workpiece.

(d). Even if a machine is operative, it cannot process workpieces if non are available to it or if there is N +1 workpieces in the storage containing one on machine 2. In the former condition, the machine 2 is said to be **starved,** in the latter machine 1 is called **blocked.** It is also assumed that the first machine is never starved and the second is never blocked.

(e). Blocked or starved machines, because they are not operating, are not vulnerable to failure.

(f). Each machine has a repairable facility. When a machine fails, it can be repaired immediately and the workpieces currently being processed must wait for the continuing processing. As soon as the repair of the failed machine is completed, the machine continues to process the work piece that just stoped. The processing time of the work piece is cumulative.

(g). All above random variables are mutually independent.

3. The Solution of Queuing System $C_2 / G / 1 / N$

For researching the model of CIMS transfer line described above, we first consider the queueing system C_2 / G / 1 / N, the descriptions of which are as following.

(a). The inter arrival times of successive customers are i.i.d., their common distribution is a Cox-distribution, i.e., a customer need one or two phases, denoted by 1 and 2, to come to the server. For passing phase 1, he will spend a random time, which is exponentially distributed with rate λ_1. After passing the phase 1, he can directly enter the server with probability p. or go to the phase 2 with probability $q = 1 - p$. On phase 2, he will spend a random time which is exponentially distributed with rate λ_2. After passing the phase 2, he directly enters the server.

(b). The amount of queuing length at most is N+1, which containing a customer who just being served on the server. When the queuing length is N+1, the customer who has passed the *arrival process* cannot enter the system because there is no room to stay. In this case, there is no other customer in the inter arrival process. Only when the queuing length is smaller than N+1, a customer can come to the system in accordance with a Cox- distribution described as (a).

(c). The service times of successive customers are i.i.d., their common distribution is a general continues distribution with mean $1 / \mu_1$ and the distribution function of which is denoted by

$$G(t) = \int_0^\infty g(x)dx = 1 - \exp\left\{-\int_0^\infty \mu(x)dx\right\}, \qquad t \geq 0.$$

Define some states as following

(0, i), i =1, 2, represent that there is no customer in the system and the customer who is coming to the server is in i –th phase of the interracial processes.

(n, i), i =1, 2, n = 1, 2,…, N+1, represent that the customer who is coming to the server is in i – th phase of the interracial processes and there are n customer in the system where a customer is being served while the others n–1 customer are waiting for service.

N+2, represents that there are N+ 1 customer in the system where a customer is being served while the others N customers are waiting for service. And a customer has passed the *arrival process,* but he cannot enter the system because there is no room to stay. In this case, there is no other customer in the interracial processes.

Letting S(t) be the system state at time t, and X (t) mean the elapsed service time of the customer currently being served at time t, we know that {S (t), (X (t)} forms a vector Markov process. Denoting by

$$p_{0i}(t) = p\{S(t) = (0,i)\}$$
$$p_{ni}(t,x)dx = p\{S(t) = (n,i), \quad x < X < x + dx\};$$
$$p_{N+2}(t,x)dx = p\{S(t) = N+2, \quad x < X < x + dx\}.$$

and

$$p_{0i} = \lim_{t\to\infty} p_{0i}(t), \qquad p_{ni}(x) = \lim_{t\to\infty} p_{ni}(t,x), \qquad p_{N+2}(x) = \lim_{t\to\infty} p_{N+2}(t,x),$$

where n =1, 2,…, N+1 and i =1, 2, then we can get the following equations and the results (1). N=0.

In this case, we only have five states (0,1), (0,2), (1,1), (1,2), and state 2. The probabilities governing the system will satisfy.

$$\lambda_1 p_{01} = \int_0^{\infty} p_{11}(x)\mu(x)dx$$

$$\lambda_2 p_{02} = \lambda_1 q p_{01} + \int_0^{\infty} p_{12}(x)\mu(x)dx$$

$$\left[\frac{d}{dx} + \lambda_1 + \mu(x)\right] p_{11}(x) = 0$$

OR $$\left[\frac{d}{dx} + \lambda_2 + \mu(x)\right] p_{12}(x) = q\lambda_1 p_{11}(x)$$

$$\left[\frac{d}{dx} + \mu(x)\right] p_2(x) = p\lambda_1 p_{11}(x) + \lambda_2 p_{12}(x)$$

$$p_{11}(0) = p\lambda_1 p_{01} + \lambda_2 p_{02} + \int_0^{\infty} (x)\mu(x)dx$$

$$p_{12}(0) = p_2(0) = 0$$

From these equations, we can readily know

$$p_{11}(x) = e^{-\lambda_1 x}\overline{G}(x)\,C$$

$$p_{12}(x) = \frac{q\lambda_1 \overline{G}(x)}{\lambda_1 - \lambda_2}\left[e^{-\lambda_2 x} - e^{-\lambda_1 x}\right]C$$

$$p_2(x) = \frac{\overline{G}(x)}{\lambda_1 - \lambda_2}\left[(\lambda_1 - \lambda_2) + (\lambda_2 - p\lambda_1)e^{-\lambda_1 x} - q\lambda_1 e^{-\lambda_2 x}\right]C$$

$$p_{01} = \frac{g^*(\lambda_1)}{\lambda_1}C$$

$$p_{02} = \frac{q}{\lambda_1(\lambda_1 - \lambda_2)}\left[\lambda_1 g^*(\lambda_2) - \lambda_2 g^*(\lambda_1)\right]C$$

where $g^*(*)$ represents Lap lace transform of a function g (*), and constant C can be gotten by normalizing condition as following.

$$C^{-1} = \frac{1}{\mu} + \frac{\lambda_1 q g^*(\lambda_2)}{\lambda_2(\lambda_1 - \lambda_2)} + \frac{(p\lambda_1 - \lambda_2)g^*(\lambda_1)}{\lambda_1(\lambda_1 - \lambda_2)}$$

(2).$1 < N < \infty$

In this general case, the equations are

$$\lambda_1 p_{01} = \int_0^\infty p_{11}(x)\mu(x)dx \tag{3.1}$$

$$\lambda_2 p_{02} = q\lambda_1 p_{01} + \int_0^\infty p_{12}(x)\mu(x)dx \tag{3.2}$$

$$\left[\frac{d}{dx} + \lambda_1 + \mu(x)\right] p_{n1}(x) = p\lambda_1 p_{n-1}(x) + \lambda_2 p_{n-12}(x) \qquad 1 \le n \le N+1 \tag{3.3}$$

$$\left[\frac{d}{dx} + \lambda_2 + \mu(x)\right] p_{n2}(x) = q\lambda_1 p_{n1}(x) \qquad 1 \le n \le N+1 \tag{3.4}$$

$$\left[\frac{d}{dx} + \mu(x)\right] p_{N+2}(x) = p\lambda_1 p_{n+11}(x) + \lambda_2 p_{n+12}(x) \tag{3.5}$$

$$p_{11}(0) = p\lambda_1 p_{01} + \lambda_2 p_{02}(x)\mu(x)dx \tag{3.6}$$

$$p_{n1}(0) = \int_0^\infty p_{n+11}(x)\mu(x)dx \qquad 2 \le n \le N+1 \tag{3.7}$$

$$p_{n2}(0) = \int_0^\infty p_{n+12}(x)\mu(x)dx \qquad 1 \le n \le N \tag{3.8}$$

$$p_{N+21}(x) = p_{N+2}(x) \text{ and } p_{N+12}(0) = p_{N+2}(0) = p_{01}(x) = p_2(x) = 0 \tag{3.9}$$

Denoting

$$I = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \cdots & \\ & & & 1 \end{bmatrix} \qquad I_0 = \begin{bmatrix} 0 & & & & \\ 1 & 0 & & & \\ & & 1 & 0 & \\ & & & \cdots & \\ & & & 1 & 0 \end{bmatrix}$$

$$p_{n1}(x) = G(x)\mathrm{f}_{n1}(x) \quad (n = 1,2,\ldots,N+1,\ i = 1,2,), \quad A = \begin{bmatrix} -\lambda_1 I + p\lambda_1 I_0 & \lambda_2 I_0 \\ q\lambda_1 I & -\lambda_2 I \end{bmatrix}$$

$$\mathrm{f}_1(x) = (\mathrm{f}_{11}(x), \mathrm{f}_{21}(x), \ldots, \mathrm{f}_{N+11}(x))' \ , \mathrm{f}_2(x) = (\mathrm{f}_{12}(x), \mathrm{f}_{22}(x), \ldots, \mathrm{f}_{N+12}(x))'$$

from equations (3.3) and (3.4), we shall have

$$\begin{bmatrix} \mathrm{f}_1'(x) \\ \mathrm{f}_2'(x) \end{bmatrix} = A \begin{bmatrix} \mathrm{f}_1(x) \\ \mathrm{f}_2(x) \end{bmatrix}$$

Since $|sI - A| = 0$ implies $(s+\lambda_1)^{N+1}(s+\lambda_2)^{N+1} = 0$, we know that there exist polynomial function $a_{ni}(x)$ and $b_{ni}(x)$ $(i = 1,2, n = 1,2,\ldots, N+1)$, which powers at most are N, such that

$$p_{n1}(x) = \overline{G}(x)\left[a_{n1}(x)e^{-\lambda_1 x} + b_{n1}(x)e^{-\lambda_2 x}\right]$$
$$p_{n2}(x) = \overline{G}(x)\left[a_{n2}(x)e^{-\lambda_1 x} + b_{n2}(x)e^{-\lambda_2 x}\right]$$

and then

$$p_{N+2}(x) = \overline{G}(x)\left\{\int_0^x e^{-\lambda_1 x}\left[p\lambda_1 a_{N+11}(t) + \lambda_2 a_{N+12}(t)\right]dt + \int_0^x e^{-\lambda_2 t}\left[p\lambda_1 b_{N+11}(t) + \lambda_2 b_{N+12}(t)\right]dt\right\}$$

$$p_{01} = \frac{1}{\lambda_1}\left[\int_0^\infty e^{-\lambda_1 x} a_{11}(x)g(x)dx + \int_0^\infty e^{-\lambda_2 x} b_{11}(x)g(x)dx + \right]$$

$$p_{02} = \frac{1}{\lambda_2}\left\{\int_0^\infty e^{-\lambda_1 x}\left[qa_{11}(x) + a_{12}(x)\right]g(x)dx + \int_0^\infty e^{-\lambda_2 x}\left[qb_{11}(x) + b_{12}(x)\right]g(x)dx\right\}$$

where the coefficients of polynomial function $a_{ni}(x)$ and $b_{ni}(x)$ can be obtained recursively by using equation (1.6)-(1.9), the normalizing conditions and the recurrence equations

$$a'_{n1}(x) = p\lambda_1 a_{n-11}(x) + \lambda_2 a_{n-12}(x)$$
$$b'_{n1}(x) = (\lambda_1 - \lambda_2)b_{n-11}(x) + p\lambda_1 b_{n-11}(x) + \lambda_2 b_{n-12}(x)$$
$$a'_{n2}(x) = \lambda_1 a_{n2}(x) + q\lambda_1 a_{n1}(x)$$
$$b'_{n2}(x) = \lambda_2 b_{n2}(x) + q\lambda_1 b_{n1}(x)$$

where $1<n<N+1$ and $a_{01}(x) = a_{02}(x)b_{01} = b_{01}(x) = b_{02}(x) = 0$.

Remark 3.1 Since $a_{ni}(x)$ and $b_{ni}(x)$ are all polynomial functions, then $\int_0^\infty p_{ni}(x)\mu(x)dx$, $\int_0^\infty p_{n+2}(x)\mu(x)dx$, and p_{0i} $(i = 1,2,\ldots, N+1)$ can be expressed by using $g^*(*)$. This result is very useful in section 1.4.

(3). $N = \infty$

In this case, the equations are

$$\lambda_1 p_{01} = \int_0^\infty p_{11}(x)\mu(x)dx \tag{3.10}$$

$$\lambda_2 p_{02} = \lambda_1 q p_{01} + \int_0^\infty p_{12}(x)\mu(x)dx \tag{3.11}$$

$$\left[\frac{d}{dx}+\lambda_1+\mu(x)\right]p_{n1}(x)=p\lambda_1 p_{n-11}(x)+\lambda_2 p_{n-12}(x) \qquad n=1,2,\ldots, \tag{3.12}$$

$$\left[\frac{d}{dx}+\lambda_2+\mu(x)\right]p_{n2}(x)=q\lambda_1 p_{n1}(x)+\lambda_2 p_{n-12}(x) \qquad n=1,2,\ldots, \tag{3.13}$$

$$p_{11}(0)=p\lambda_1 p_{01}+\lambda_2 p_{02}+\int_0^\infty p_{21}(x)\mu(x)dx \tag{3.14}$$

$$p_{n1}(0)\int_0^\infty p_{n+11}(x)\mu(x)dx \qquad n=2,3,\ldots, \tag{3.15}$$

$$p_{n2}(0)\int_0^\infty p_{n+12}(x)\mu(x)dx \qquad n=1,2,\ldots, \tag{3.16}$$

$$p_{01}(x)=p_{02}(x)=0 \tag{3.17}$$

For discussing convenience, we introduce some notations as following

$$p_1(z,x)=\sum_{n-1}^{\infty}z^n p_{n1}(x),\quad p_2(z,x)=\sum_{n-1}^{\infty}z^n p_{n2}(x),\quad p(z,x)=(p_1(z,x),\ p_2(z,x))'$$

$$s_1(z)=\frac{1}{2}\left[\lambda_1(pz-1)-\lambda_2+\sqrt{(\lambda_1+\lambda_2-\lambda_1 pz)^2+4\lambda_1\lambda_2(z-1)}\right]$$

$$s_2(z)=\frac{1}{2}\left[\lambda_1(pz-1)-\lambda_2-\sqrt{(\lambda_1+\lambda_2-\lambda_1 pz)^2+4\lambda_1\lambda_2(z-1)}\right]$$

$$T=\begin{bmatrix}\lambda_2+s_1(z) & \lambda_2+s_2(z)\\ \lambda_1 q & \lambda_1 q\end{bmatrix},\quad T^{-1}=\frac{1}{q\lambda_1(s_1(z)-s_2(z))}\begin{bmatrix}\lambda_1 q & -\lambda_2-s_2(z)\\ -\lambda_1 q & \lambda_2+s_2(z)\end{bmatrix}$$

Lemma 1.1

$$s_1(1)=\lim_{n\to t}s_1(z)=0\ ,\quad s_2(1)=\lim_{n\to t}s_2(z)=-\lambda_2-q\lambda_1.$$

$$s_1'(1)=\lim_{n\to t}s_1'(z)=\frac{\lambda_1\lambda_2}{\lambda_2+q\lambda_1}\ ,\quad s_2'(1)=\lim_{n\to t}s_2'(z)=\frac{q\lambda_1(p\lambda_1-\lambda_2.)}{\lambda_2+q\lambda_1}$$

Proof: This is straightforward from the definitions of $s_1(z)$ and $s_2(z)$.

According to the equations (3.12) and (3.13), by using the notations above, we can get

$$p(z,x)=\overline{G}(x)T\begin{bmatrix}e^{s_1(z)x} & 0\\ 0 & e^{s_2(z)x}\end{bmatrix}C$$

$$p(z,x)=\overline{G}(x)T\begin{bmatrix}e^{s_1(z)x} & 0\\ 0 & e^{s_2(z)x}\end{bmatrix}T^{-1}p(z,x) \tag{3.18}$$

Substituting (3.18) into (3.15) and (3.16), noting (3.10), (3.11), and (3.14), we have

$$p(z,0) = zT\begin{bmatrix}\left[z - g^*(-s_1(z))\right]^{-1} & 0 \\ 0 & \left[z - g^*(-s_2(z))\right]^{-1}\end{bmatrix}T^{-1} \cdot \begin{bmatrix}\lambda_1(zp-1) & \lambda_2 z \\ \lambda_1 q & -\lambda_2\end{bmatrix}\begin{bmatrix}p_{01} \\ p_{02}\end{bmatrix}$$

and then from (3.18), we get

$$p(z,x) = z\overline{G}(x)T\begin{bmatrix}e^{s_1(z)x}\left[z - g^*(-s_1(z))\right]^{-1} & 0 \\ 0 & e^{s_2(z)x}\left[z - g^*(-s_2(z))\right]^{-1}\end{bmatrix}T^{-1}$$
$$\cdot\begin{bmatrix}\lambda_1(zp-1) & \lambda_2 z \\ \lambda_1 q & -\lambda_2\end{bmatrix}\begin{bmatrix}p_{01} \\ p_{02}\end{bmatrix} \tag{3.19}$$

By using Lemma 3.1, we have

$$p_1(z,x) = \sum_{n-1}^{\infty}\int_0^{\infty}\left[p_{n1}(x) + p_{n2}(x)\right]dx = p_1(1,x) + p_2(1,x) = ep(1,x)$$
$$= \frac{\overline{G}(x)}{\lambda_1 q}\lim_{n\to t} s_1\left[z - g^*(z)\right]^{-1}(\lambda_1 q, -\lambda_2 - s_2(z))\begin{bmatrix}\lambda_1(zp-1) & \lambda_2 z \\ \lambda_1 q & -\lambda_2\end{bmatrix}\begin{bmatrix}p_{01} \\ p_{02}\end{bmatrix}$$
$$= \frac{\lambda_1\lambda_2\overline{G}(x)}{\lambda_2 + q\lambda_1 - \frac{1}{\mu}\lambda_1\lambda_2}(p_{01} + p_{02}) \tag{3.20}$$

Therefore by using normalizing condition, we have

$$p_{01} + p_{02} = \frac{\lambda_2 + q\lambda_1 - \frac{1}{\mu}\lambda_1\lambda_2}{\lambda_2 + q\lambda_1}(p_{01} + p_{02}) \tag{3.21}$$

and then from (3.20), we have

$$\sum_{n-1}^{\infty}\int_0^{\infty}\left[p_{n1}(x) + p_{02}(x)\right]dx = p_1(1,x) + p_2(1,x) = ep(1,x) = \frac{\lambda_1\lambda_2\overline{G}(x)}{\lambda_2 + q\lambda_1} \tag{3.22}$$

4. Performance Measures of the Transfer Line

In this section, we shall discuss the performance measures of the CIMS transfer line described in section 2.

Let X_i represent the processing completion time of a work piece on machine i (i=1, 2), that is actual processing time plus the delay time because of machine failure, we have

Theorem 4.1. Denote $C(t) = P(X_1<t)$, $G(t) = P(X_2<t)$, then

$$(1).\ g^*(s) = f^*\left[s + \alpha_2 - \alpha_2 r^*(s)\right],\ \text{and}\ EX_2 = \frac{1}{\mu} = \frac{1}{\mu}(1 + \frac{\alpha_2}{\beta_2}).$$

$$(2).\ \overline{C}(t)\frac{1}{\lambda_2 - \lambda_1}\left[(\lambda_2 - \mu_1)e^{-\lambda_1 t} - (\lambda_1 - \mu_1)e^{-\lambda_2 t}\right]$$

where

$$\lambda_1 = \frac{1}{2}\left[\alpha_1 + \beta_1 + \mu_1 + \sqrt{(\alpha_1 + \beta_1 + \mu_1)^2 - 4\beta_1\mu_1}\right]$$

$$\lambda_2 = \frac{1}{2}\left[\alpha_1 + \beta_1 + \mu_1 + \sqrt{(\alpha_1 + \beta_1 + \mu_1)^2 - 4\beta_1\mu_1}\right]$$

Proof: According to reference [6], result (1) is straightforward. And then by using the result (1), we can readily get result (2) when the corresponding distributions are exponentially distributed.

Next, we consider a special CIMS transfer line consisting of two reliable machines, the processing time of a work piece on machine i is exponentially distributed with rate λ_i (i = 1,2). After processing by machine 1 , a work piece will directly exist the CIMS transfer line with probability p, and otherwise go to machine 2 for continuing processing with probability q =1-p. After being processed by machine 2, a work piece exist the CIMS transfer line. For this special CIMS transfer line, if we denote by Y the complete processing time of a work piece on the CIMS transfer line, we have

Theorem 4.2

$$p(Y > t) = \frac{1}{\lambda_2 - \lambda_1}\left[(\lambda_2 - \mu_1)e^{-\lambda_1 t} - (\lambda_1 - \mu_1)e^{-\lambda_2 t}\right]$$

Proof: If we denote by state i (i = 1,2) the case when a work piece is being processed on machine i at time t, and that the case when the work piece exit the CIMS transfer line is an absorbing state, we shall get a Markov process with absorbing state, and the states probabilities $P_1(t)$ and $P_2(t)$ satisfy

$$\left[\frac{d}{dt} + \lambda_1\right]p_1(t) = 0$$

$$\left[\frac{d}{dt} + \lambda_2\right]p_2(t) = q\lambda_1 p_1(t)$$

$$p_1(0) = 1, \quad p_2(0) = 0$$

From these, we easily obtain

$$p_1(t) = e^{-\lambda_1 t}\ , \qquad p_2(t) = \frac{q\lambda_1}{\lambda_1 - \lambda_2}\left[e^{-\lambda_2 t} - e^{-\lambda_1 t}\right]$$

Therefore

$$p(Y-t) = p_1(t) + p_2(t)$$

$$p(Y>t) = \frac{1}{\lambda_2 - \lambda_1}\left[(\lambda_2 - p\lambda_1)e^{-\lambda_1 t} - q\lambda_1 e^{-\lambda_2 t}\right]$$

According to the theorem 4.1 and 4.2, we know that if we define P-μ_1 / λ_1, the CIMS transfer line described in section 2, is equivalent to queuing system C_2 / G / 1 / N described in section 3, from the viewpoint of performance measures such as production rate, blocked probability and starved probability. And then by using the results in section 3, we can obtain the performance measures of CIMS transfer line as following

(a). By using the general formula [7] , we know that the production rate of the CIMS transfer line, denoted by W_N , is given by

$$W_N = \sum_{n=1}^{N+1}\int_0^\infty [p_{n1}(x) + p_{n2}(x)]\mu(x)dx + \int_0^\infty p_{n+2}(x) + \mu(x)dx$$

(b). According to the definitions of the blocked probability and the starved probability described in section 2, we have

$$p_B = \int_0^\infty p_{n+2}(x) + dx \qquad p_S = p_{01} + p_{02}$$

Expressing clearly, we get

(1). When N = 0

$$W_0 = \int_0^\infty [p_{11}(x) + p_{12}(x) + p_2(x)]\mu(x)dx$$

$$= \left[\frac{1}{\mu} + \frac{\lambda_1 q g^*(\lambda_1)}{\lambda_1 - \lambda_2} + \frac{p\lambda_1 - \lambda_2}{\lambda_1(\lambda_1 - \lambda_2)} g^*(\lambda_1)\right]^{-1}$$

$$p_B = \int_0^\infty p_2(x)dx\left[\frac{1}{\mu} + \frac{(\lambda_2 - p\lambda_1)[1 - g^*(\lambda_1)]}{\lambda_1(\lambda_1 - \lambda_2)} - \frac{q\lambda_1[1 - g^*(\lambda_2)]}{\lambda_2}\right]W_1$$

$$p_S = p_{01} + p_{02} = \left[\frac{g^*\lambda_1}{\lambda_1} + \frac{q\lambda_1 g^*[1 - g^*(\lambda_2) - \lambda_2 g^*(\lambda_1)]}{\lambda_2}\right]W_1$$

(2). when $1 < N < \infty$.

$$W_N = \sum_{n=1}^{N+1}\int_0^\infty [p_{n1}(x) + p_{n2}(x)]\mu(x)dx + \int_0^\infty p_{n+2}(x) + \mu(x)dx$$

$$p_B = \int_0^\infty p_{n+2}(x) + \mu(x)dx \qquad p_S = p_{01} + p_{02}$$

where $p_{ni}(x)$, $p_{N+2}(x)$, p_{0i} (i = 1,2, n = 1,2,..., N+1), are given by (2) in section 1.3.

(3). When N = ∞.

$$W_\infty = \int_0^\infty \sum_{n=1}^{\infty} [p_{n1}(x) + p_{n2}(x)]\mu(x) = \int_0^\infty p_1(1,x) + p_2(1,x)\mu(x)dx = \frac{\lambda_1\lambda_2}{\lambda_2 + q\lambda_1}$$

$$p_B = 0, \qquad p_S = p_{01} + p_{02} = \frac{\lambda_2 + q\lambda_1 - \frac{\lambda_1\lambda_2}{\mu}}{\lambda_2 + q\lambda_1}$$

where λ_1, λ_2, $1/\mu$ and g^* (*) are given by theorem 4.1.

Conclusion

According to the results above, we know that when N = ∞, the performance measures are only relevant to the expectations of corresponding random variables, and are no related to the general distributions. However, when 1< N < ∞, the results depend closely on the general distributions.

According to the idea, of this section, we know that if processing times and repair times of two machines are all general distributions, and the lifetimes of which are all still exponentially distributed, then by using theorem 1 the CIMS transfer line is equivalent to a queuing system GI / G / 1 / N from the viewpoint of production rate, blocked probability and starved probability.

References

[1]. Gershwin, S.B and O. Berman (1981). Analysis of transfer lines consisting of two unreliable machines with random processing times and finite storage buffers AIIE Trans 13, 2-11.

[2]. Gershwin, S.B and Schicle, I.C (1980). Modeling and Analysis of two and three-stage transfer lines with random processing times and finite storage buffers, MIT Laboratory for information and decision systems. Report LIDS-R- 979.

[3]. Shu, S.G (1992). Analysis of repairable CIMS with buffers and a study of system reliability, Acta Auto Sin; 18, 15-22.

[4]. Shu, S.G and Tan, M. (1991). Reliability analysis of local network in CIMS. Institute of Automation, Chinese Academy of Science.

[5]. Zhang, Y.G. (1991). Reliability Modeling and Analysis of CIMS transfer line, Ph.D Thesis, Institute of Automation, Chinese Academy of Science, Beiging.

[6]. Cao, J.H and Cheng, K. (1982). Analysis of M/G/1 Queuing system with repairable service station, Acta Math. Appl. Sin, 5, 113-127.

[7]. Shi, D.H (1985). A new method for Calculating the mean failure number of a repairable system during (o,t], Acta Math., Appl. Sin; 8, 101-110.