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THE MATHEMATICS OF MACHINE REPAIR PROBLEM

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Abstract

The paper attempts to discuss mathematical methods and techniques, including queue theory, Markov Chains and simulation to formulate and solve problems concerned with machine repair.

Keywords: Mathematical methods and techniques.

1. Introduction

The term machine repair (or interference) is generally reserved for a unique type of queuing problem when failed machines wait for repairs. Production is lost whenever a machine is stopped, whether it is being repaired or waiting to be repaired.

In a number of industrial processes, servers (operators) are required to attend a number of Semi-automatic or automatic machines. Let a server be in charge of a number of identical machines, which stop from time to time. When a machine stops a server repairs it and sets it running again. This repair will take finite time and during this time a second or third machine might stop, and this will have to remain stopped until the server is free to attend it. Thus the production will be lost, various mathematical models of the system have been proposed to predict this loss.

Two quantities of fundamental importance in the context of machine repair problem, abbreviated as MRP, are the machine availability (or machine efficiency) and the operative (server) utilization. The machine availability (MA) is defined by Benson and Cox (1951). as the fraction of the total time that the machines are working. The operative utilization (OU) is the proportion of time the server is working on repairs.

Machine repair is a unique type of queuing problem. Since there is only a finite number of customers (machines) in the system, the arrival (stoppage) rate of machines is dependent on the number of machines running. Since stoppage in a manufacturing system occurs randomly, it is difficult task for management to determine economically optimal assignment of machines to the server. By decreasing the number of machines per server, the amount of server idle time is increased. By increasing the number of machines per server, the machine availability decreases due to increased down time. Therefore, an optimal solution must be achieved between server idle time and machine availability.

To tackle this situation, cost models in MRP have been studied by various authors including Albright (1980) Ashcroft (1950), Morse (1958), Elsayed (1981) and Burney et al (1994).

The MRP with cold standbys (a standby component whose failure rate is zero is called cold standby) were first considered by Taylor and Jackson (1954) while the incorporation of a cost model for cold standbys was studied by Gross, Kahn and Marsh (1977) and Hilliard (1976). More recently, cost models applied to related problems in reliability were considered by Geol, Gupta and Singh (1981). Murari and Goyal (1985) and Singh and Srinivasu (1987). The M/M/R warm standby (a standby component whose failure rate is non-zero and is less than the failure

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rate of an operating machine, is called the warm standby) model generalizes the well studied cold-standby model in Sivizlian and Wang (1989).

The paper is organized as follows:

In section 2 The 'Perfect' M/M/1 Machine Repair Problem due to Palm is discussed. Queuing systems formatted as patrolling machine repair problem is given in section 3.Section 4 deals with simulation and detailed discussion on its languages. Finally conclusion is made in section 5.

2.The ' perfect' M/M/1 Machine Repair Problem

The 'perfect' M/M/1 Machine Repair Problem, which is described as the basic system for machine repair is a system with one server attending M machines. In the solution of this problem Palm (1947) makes the following model assumptions:

1. All machines were considered to have the same average rate for stoppage and service time.
2. Stoppages occurred randomly according to the Poisson distribution with mean σ.
3. The service times were of random length, which was described by the exponential distributions with mean $1/\mu$.
4. All random variables were independently distributed
5. The system of machines and server was in a state of statistical equilibrium.
6. The server had no other duties except to serve the machines under his care. Walking time to a stopped machine was considered negligible and, therefore, ignored.
7. The order that stopped machine were serviced was FIFO (First in, first out), that is, machines were repaired in the same order that they stopped.

To describe Palm's solution it is necessary to understand the Poisson distribution. The general formula of the Poisson distribution is $P_n(t)=[(\sigma t)^n \exp(-\sigma t)/(n!)$, where $P_n(t)$ is the probability of n machines stopping during the time t. From this equation it is easy to derive the following probabilities:

1. The probability that no stoppages occur during the time t(n=0) is equal to
   $P_o(t)=\exp(-\sigma t)$.
2. The probability that one stoppage occurs during the time t(n=1) equals
   $P_1(t) = \sigma t \exp(-\sigma t)$.

Knowing the above two probabilities and the normalized condition $\Sigma_{n=o}P_n(t)=1$, the probability of more than one stoppages occurring during t time can be derived as follows:

$\Sigma^{\infty}_{n=o}[P_n(t)-P_o(t)-P_1(t)]=1-\exp(-\sigma t)-\sigma t\exp(-\sigma t)$

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$$=1-[\ 1-\sigma t+(\sigma t)^2/2!-(\sigma t)^3/3!+\ldots\ ]$$

$$-\sigma t[\ 1-\sigma t+(\sigma t)^2/2!-(\sigma t)^3/3!+\ldots]$$

$$=(\sigma t)^2/2!-(\sigma t)^3/3!+\ldots=o\ (t^2)$$

The term $o(t^2)$ is a function of t^2; and for very small values of t, this function approaches zero since terms are power of t. Therefore, the probability of more than one stoppage during a small time interval (o,t) is negligible and can be considered zero. The $o(t^2)$ term can be used for functions of t that are considered negligible. Using this convention the following probabilities are derived:

1. The probability of at least one stoppage during the interval (0,t) is
   $\Sigma^{\infty}_{n=o}[\ P_n(t)-P_o(t)\ ]=1-\exp(-\sigma t)=1-[1-\sigma t+o\ (t^2)\ ]$
   $=\sigma t$
2. The probability of no stoppages during the interval (0,t) is
   $P_o(t)=\exp(-\sigma t)=1-\sigma t+o\ (t^2)$
   $=1-\sigma t$

Now the probabilities of entering a state during the small time interval (t, t+h) ar examined. If no stoppages were in the system at time t, the probability of having no stoppages at time (t+h) is

$$P_o\ (t+h)=P_o(t)\ P_o(h)=P_o(t)\ (1-\sigma h)$$

that is, $P_0(t+h)$ is equal to the product of the probabilities of two independent events ($P_0(t)$ and $P_0(h)$). Now looking at the case of $n\geq 1$ stoppages by the time (t+h), the following probability equation results:

$$P_n(t+h)=P_n(t)\ P_o(h)+P_{n-1}(t)\ P_1(h)+o\ (t^2)$$

$$=P_n(t)\ (1-\sigma h)+P_{n-1}(t)\ \sigma h$$

The $o(t^2)$ term represents the arrival of more than one stoppage during interval h and can be ignored.

Thus, for only the arrival of stoppages into the system have been observed. This is known as a "pure birth" process. Now consider the probability of machines leaving the system; that is, machines finishing their service and returning to the running state. When a system has both arrivals (stoppages) and departures (clearings) of machines, it is referred to as the "birth-death" process. Feller (1950) described both the "birth-death" process and the "pure birth" process. In Feller's notation E_n represents the state of the system with n machines stopped. Therefore, the "Pure birth" process allows only changes of the type E_n to E_{n+1}. Departures from the system are represented by a state change of E_n to E_{n-1}. Using this notation the state E_n can be reached at time (t+h) only if one of the following four conditions are satisfied:

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1. The system is in the state E_n at time t and there are no arrivals or departures during the interval h. The probability of this occurring is equal to

   $1-\sigma_n h)\ (1-\mu_n h)\ P_n(t)$

2. With the system in the state E_{n-1} at time t. There was one arrival during the interval h. The probability of this occurrence is

   $\sigma_{n-1} h\ (1-\mu_{n-1} h) P_{n-1}(t)$

3. At time t the system's state is E_{n+1} and there is one departure from the system during h. The probability of this taking place is

   $(1-\sigma_{n+1} h\ (\mu_{n+1} h)\ P_{n+1}(t)$

During the small interval h there are a multiple number of arrivals and for departures which occur. Therefore, the system starts in the state E_n at t and ends in the state E_n at (t+h). The probability of this occurring is negligible and equal to $o(t^2)$. Therefore, the probability of being in the state E_n at time (t+h) is equal to the sum of the above four probability cases.

$$P_n(t+h) = P_n(t)\ (1-\sigma_n h)\ (1-\mu_n h)$$

$$+P_{n-1}(t)\ (\sigma_{n-1} h)\ (1-\mu_{n-1} h)$$

$$+P_{n+1}(t)\ (1-\sigma_{n+1} h)\ (\mu_{n+1})+o(t^2), \text{ for } n>o$$

The probability of the system being in the state E_n at time (t+h) is then equal to

$$P_n(t+h) = P_n(t)\ [1-(M-n)\ \sigma h\ ](1-\mu h)$$

$$+P_{n-1}(t)\ [(M-n+1)\ \sigma h]\ (1-\mu h)$$

$$+P_{n+1}(t)\ [1-(M-n-1)\sigma h](\mu h)$$

$$+0(t^2),\ \text{for } 0<n<M \qquad (1)$$

This probability equation was derived from the four probability cases stated above.

Examining the boundary line conditions (n=0, n=M, the following two probability equations result:

$$P_o(t+h) = P_o(t)\ (1-M\sigma h)+P_1(t)\ (\mu h), \qquad (2)$$

$$P_M(t+h) = P_M(t)\ (1-\mu h) +P_{M-1}(t)\ (\mu h), \qquad (3)$$

Thus equations (1), (2) and (3) take the form

$$-P_n[\ (M-n)\ \sigma+\mu\ ]+P_{n-1}(M-n+1)\ \sigma+P_{n+1}\mu = 0$$

$$\text{for } 0<n<M., \qquad (4)$$

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$$-P_o M\sigma + P_1\mu = 0, \qquad \text{for } n = 0 \tag{5}$$

and

$$-P_M\mu + P_{M-1}\sigma = 0, \qquad \text{for } n = M \tag{6}$$

A numerical example of Palm's Model is shown in table1.

Table. 1

Perfect* M/M/1 Model
(Probabilities P_n for the case σ=0.2, M=5)

n	P_n/P_o	P_n
0	1.0	0.2849
1	1.0	0.2849
2	0.8	0.2279
3	0.48	0.1367
4	0.192	0.0547
5	0.0384	0.0109
	3.5104	1.0000

$1/P_o = \Sigma P_n/P_o = 3.5104$

$\therefore P_o = 1/3.5104 = 0.2849$

Operative utilization = $1-P_o = 0.7151$
Machine availability = $0.7151/My = 0.7151/[5(0.2)] = 0.7151$

We are interested in such queuing system which can be formatted as MRP system. In this section first we will consider MRP with random breakdown of machines and general identically distributed service (repair) times. Ashcroft (1950) solved the server (operative) utilization by computing the average duration of the periods for which the server is free and busy. His expression for average number of machines running in the case of exponentially distributed repair times, and constant repair times are:

$$E[M] = \frac{MYn}{1+MyYn}, \tag{7}$$

Where y is servicing factor and

$$Y_M = 1+ (M-1)\, y+ (M-1)\,(M-2)y^2+ \ldots (M-1)!y^{M-1} \tag{8}$$

in the case of exponentially distributed times, and

$$Y_M = 1+\binom{M-1}{1}(e^y+1)\binom{M-1}{2}(e^y - 1)\,(e^{2y}-1) + \ldots + \binom{M-1}{M-1}(e^y -1)\ldots e^{(M-1)y} - 1,$$

for constant repair times.

Table 2 compares the average number of machines running in these two cases. The values for exponential repair times are lower than the corresponding values for constant repair times, consistent with the well known phenomenon in the queueing theory, that the queue length (which causes production loss in this case) increases with the variance of service time (repair time).

Table. 2

Average Number of Machines Running
Repair times

y	Exponential	Constant
0.02	9.76	9.78
0.04	9.44	9.52
0.06	9.01	9.19
0.08	8.48	8.76
0.10	7.85	8.21
0.30	3.33	3.33
0.50	2.32	2.32

one can see that the differences between these values (exponential and constant repair times) are quite small. Thus although the two repair time distributions differ radically the final outcomes are similar (for higher values of y)., one advantage of exponentially distributed repair times is that it is relatively easy task to extend the model to deal with a situation where a team of R servers attend the M machines.

3. Directional patrolling system

The work so far described does not take into account the time that the server spends in walking to broken down machines. This is assumed to be zero. Such an assumption will not always be justifiable, and in these cases there is added point that it may be desirable for the server to patrol the machines in some systematic way, repairing broken machines, and checking and maintaining others. Attempts to use the methods describes earlier in his paper (Brunnschweiler, 1954)., by 'suitably' modifying the repair time distribution proved unsuccessful and the first solution published is that by Mack et al (1957). They give tables of the machine availability, in case of random breakdowns, constant traveling time constant repair time and a uni-directional system of patrolling. Starting at the first machine, the server inspects and repairs it (if necessary); he then proceeds to the second machine, then to the third and so on, eventually returning to the first machine when the whole cycle begins again.

Bunday and Mack (1973) obtained a theoretical solution to a second type of patrolling problem. The M machines in the server's case may be thought of as being equally spaced along a line, and labeled say from left to right, 1,2,...M. starting at the first machine the server inspects and services it before travels to the next machine and so on to the end of the line. At the end of the line, the server moves back to (M-1) the machine and so returns to machine 1 when cycle begins again.

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The total time to walk from one machine to the next and inspect and service that machine is called the traveling time, and assumed to be a constant W for each machine.

The time taken to repair a broken down machine is a constant r. Breakdown of the machines are assumed to occur at random during running time at average rate σ for each machine.

It is not fruitful to describe the system in terms of the number of machines stopped at a given time; it is better to describe the situation encountered by the server as he traverses the machines.

Following the Bunday and Mack (1973), we let $\{x\mu_2\mu_3,...,\mu_M\}$ denote the left to right traverse in which, on leaving machine 1, machine i is found in state μ_i. If $\mu_i = 0$, the machine i is found running, if $\mu_i = 1$ machine i is found stopped: $P\{x\mu_2\mu_3,...,\mu_M\}$ is the probability of finding this situation on a left to right traverse. Similarly $P\{v_1v_2, \ldots, v_{M-1}x\}$ is the probability of encountering the state $\{v_1v_2, \ldots, v_{M-1}x\}$ on a right to left traverse. In the steady-state situation

$$P\{x\mu_2\mu_3, \ldots, \mu_M\} = P\{\mu_M\mu_{M-1} \ldots \mu_3\mu_2x\} \tag{10}$$

These are 2^{M-1} state probabilities. For three machines there are

$p\{x00\}, P\{x01\}, P\{x10\}, P\{x11\}$

As any left to right traverse is proceeded by one of the right to left traverses.

$$P\{x\mu_2...\mu_M\} = \sum_{\{v_1v_2...x\}} P\{v_1v_2...v_{M-1}x\} Pr\{x\mu_2...\mu_M | v_1...v_{M-1}x\} \tag{11}$$

where $Pr\{x\mu_2...\mu_M|v_1...v_{M-1}x\}$ is the conditional probability of encountering $\{x\mu_2...\mu_M\}$ immediately following

$\{v_1v_2...v_{M-1}x\}$. Thus

$$P\{x\mu_2...\mu_M\} = \sum_{\{xv'_2...v'_M\}} P\{xv'_2...v'_M\} Pr\{x\mu_2...\mu_M | v'_M...v'_2x\}... \tag{12}$$

for all $\{x\mu_2...\mu_M\}$. Also of course

$$\sum_{\{x\mu_2...\mu_M\}} P\{x\mu_2...\mu_M\} = 1 \tag{13}$$

The transition probabilities $Pr\{x\mu_2...\mu_M|v_1...v_{M-1}x\}$ are readily calculated under the assumptions stated, although the equations are complicated (Bunday and Mack, 1973). These equations constitute a set of simultaneous equations sufficient to determine the $P\{x\mu_2...\mu_M\}$. They are of course the usual equations for the steady-state probability distribution for a Markov process with the stated transition probabilities.

A knowledge of the $P\{x\mu_2...\mu_M\}$ enables the average traverse time to be calculated. This is related rather a simple way to the machine availability, A.

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In a long time T hours, a total of MAT running hours will be achieved. Thus the number of breakdowns (which equals the number of repairs) will be σ MAT. The time spent on repairs thus σ MArT, so that T-σMArt is spent on walking. Thus the number of traverses completed in time T is

$$T(1-\sigma MAr)/(M-1)W \tag{14}$$

so that average transverse time τ is

$$\tau = (M-1)W/1-\sigma MAr) \tag{15}$$

when $$A = [\tau-(M-1)W]/\sigma Mr\tau \tag{16}$$

τ is calculated simply as

$$\tau = (M-1)\,W + r\Sigma P\{\, x\mu_2...\mu_M \}(\mu_2+\mu_3+...+\mu_M) \tag{17}$$
$$\{x\mu_2...\mu_M\}$$

The two terms on the right being the walking time and average time spent on repairs, respectively.

In this way tables of the machine availability, for M=2 up to 8, and a range of values for the other parameters, are obtained. As the number of machines grows more and more, the computations of 2^{M-1} simultaneous equations become a tedious job; one way to overcome this difficulty and obtain at least good approximate solution is to simulate the system on a digital computer. In order to obtain an estimate of the machine availability which corresponds to the steady-state solutions it is necessary that the simulation extends over a long period. However, by out putting intermediate values it is possible to exhibit the convergence to the final estimate of the machine availability.

Dr. Shaikh, Noor A (1996) has done this for the system described, The simulation extended over 240 hours, each simulation run took about 30 seconds in computer's laboratory at the institute of mathematics and computer science, university of Sindh (partial work was done at Department of Computer Science, University of Karachi) The result includes up to 200 machines.

It is useful of interest to compare the results of machine availabilities between uni-directional and bi-directional patrolling systems (Mack et al, 1957 and Bunday and Mack, 1973). We have summarized their results in the table3. This shows that for the same values of the parameters, a higher value of the machine availability is obtained by adopting a system of uni-directional patrolling, that it is better to arrange the machines so that the server can patrol in uni-directionally rather than bi-directionally, to arrange say the six machines allocated to the server, as shown in figure 1(a) rather then figure1(b).

In recent years a number of authors have tackled the problem of 'non-identical' sets of machines. Among those contributing are Elasyed (1981), Kameda (1982) and Sztrik (1985). Bunday et al (1992) and Sztrik and Bunday (1993).

In real life there are many applications of 'non-identical' MRP systems, such as multi-programmed computer system or in some industry (e.g. textile industry) where old and new machines work together.

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Uni-directional Patrolling Queueing System

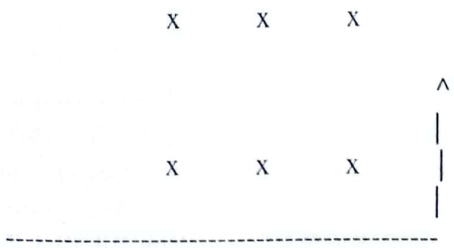


Figure.1 (a)

Bi-directional Patrolling Queueing System

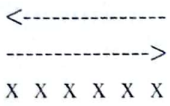


Figure 1 (b)

Further a 'non-identical' MRP can be converted in 'identical' case if we replace different groups of machines by just one group of 'identical' machines.

Table. 3

Br=0.05
MσW=0.15

M	Uni-directional Travelling time variable	Bi-directional Travelling time constant
2	0.8840	0.8774
4	0.875	0.8701
8	0 .8540	0.8639
12	0 .8240	0.8254
16	0.7840	0.7912
20	0.7320	0.7254
24	0.6710	0.6630
28	0.6090	0.6041

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Values for uni-directional and bi-directional systems are reproduced from the work of Mack et al (1957) and Bunday and Mack (1973) respectively.

4. Simulation

In an analytical solution, we can represent the behavior of our systems by sets of equations giving relationships between the variables which describe the systems. Of course in some more complex systems, we might find that we could only obtain numerical solutions to our equations. In others it may be difficult or even impossible to make the system tractable. We cannot conveniently express the relationships that govern the system in a mathematical form.

Simulation in general means using computers to imitate the activities in the system so as to study the behavior of that system. Shannon (1975) defines simulation as a process of designing a model of the real system and conducting experiments with this model to understand the behavior of that system. Simulation has been applied to such problem areas as inventory control, network analysis, production scheduling, hospitals see, for example Shaikh and Burney (1994), Fetter and Thomson (1965).

Simulation is a slow, iterative experimental problem-solving technique. Sometimes it is referred to as the method of last resort. One should contemplate problem-solving by simulation only when,

(a) The real system does not exist and it is expensive, time-consuming, hazardous, or impossible to build and experiment with prototypes (new design of a computer, solar system, nuclear reactor).

(b) There is need to study the past, present, or the future behavior of the system in real time, expanded time or compressed time (real-time control system, slow-motion studies, population growth, side-effects of new drugs).

(c) Mathematical models have no simple and practical analytical or numerical solutions (nonlinear differential equations, stochastic problems).

(d) Satisfactory validation of simulation models and results is possible.

(e) Expected accuracy (results cannot be better than input data) of simulation results is consistent with the requirements of the particular problem.

Simulation Languages

High level or procedure oriented (Payne, 1988) languages have been developed to ease the job of programming. The use of simulation as an analysis and prediction tool has led to the development of a number of languages that are simulation oriented languages. These simulation languages provide specific statements and options for representing the system at one point in mind and moving state of the system from one state to another.

The selection criteria for the useful tool is discussed by Pidd (1984). A detailed description on discrete-event simulation languages such as SIMULA; SIMSCRIPT; GPSS and GAPIV can be

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found, in Schriber (1974), in Bobillier, Kahan and Probst (1976) and by its originator, in Gordon (1975) and again, along with SIMSCRIPT, in Gordon (1978). SIMSCRIPT is described by its developers in Kiviat, Villanueva and Markowitz (1973). GASPIV is described by its originator in Pritsker (1974) and Pritsker and Pegden (1979) describe SLAM, which extends GASPIV to include queuing network models. Mention should be made of RESQ, which is a software package, consisting of an analytic part and a simulation part, that is designed specifically for the analysis of queueing networks of the type that arises in modeling for computer performance evaluation. A good survey paper on simulation with some discussion of programming languages is given in Shannon (1975).

The recent 'up-to-date' version micro-GPSS is presented by Ståhl (1990).

In a recent approach colored stochastic petrinets abbreviated as CSPN with general firing times provide building blocks for formal specification of discrete-event stochastic systems.

A CSPN is specified by a finite set of 'places', a finite number of 'transitions', and a finite set of 'colours' along with an 'input incidence function', and 'output incidence function'. The 'marking process' of a CSPN (Hass and Shedler, 1988) records the marking as it evolves over continuous time and is defined in terms of a general state space markov chain (GSSMC) that describes the net at successive making-change epochs. In the context of discrete-event stochastic systems, the marking of the CSPN corresponds to the state of the system, and the firing of a transition corresponds to the occurrence of an event. The basic features of GPSS are as follows:

1. Built-in random number generators.
2. The ability to sample from well known distributions (such as, the uniform, exponential, and normal distributions).
3. The ability to define and sample from empirical distributions.
4. A built-in simulated clock.
5. Automatic generation of queueing logic and statistics at points defined in the model.
6. The capability of assigning priority to events and transactions in the model.
7. Automatic gathering and printing of various statistics describing model behavior.

The basic skeleton of an event driven simulation can be summarized in the following algorithm:

8.2.1 Algorithm

```
        initialize;
        repeat
                set time to next event
                perform event routine
until end of simulation
report results
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5.Conclusion

Our attempts to formulate and solve problems concerned with machine repair problem have involved a variety of mathematical methods and techniques, including queue theory, Makov chains and simulation. There is still the need for improved models and more powerful methods to deal with the many problems that remain in this field. This is the challenge for the future to the interested reader.

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