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A COMPARISON OF DIFFERENT CLASSES FOR FUZZY IMPLICATION IN MODELING AN APPROXIMATE REASONING MODEL

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Abstract

Nearly after a decade after the idea given by Zadeh (1965), the theory of fuzzy sets started to provide a powerful and useful tool for the modeling of "daily life systems" which involve decision processes. The most important operator which plays a very important role for decision making in intelligent systems is the implication operator. In this paper, we study the different models of fuzzy operations of Min., Max. and Inverse, given by various researchers of Fuzzy Logic, for the construction of three classes of implication operators, giving a variety of results, showing the sensitive effects of Fuzzy operators.

In the end, a simple illustration is given, which depicts that the fuzzy systems using implications can be very effectively used in different areas such as process control, modeling, decision making, analysis, etc.

Key Words: Fuzzy set, decision process, fuzzy logic, fuzzy operators, modeling, decision making

Introduction

Implication is primarily used to mathematically approximate the rule-based models. A rule-based fact consists of *'IF ... THEN'* predicate. This gives the facility to formulate the real world problems inscribed in natural language[2], aiming at producing some inference. Fig 1. Shows the examples of a rule-based decision system.

1. **IF** (service is poor) **THEN** (tip is cheap)
2. **IF** (service is good) **THEN** (tip is average)
3. **IF** (service is excellent) **OR** (food is delicious) **THEN** (tip is generous)

Figure 1

The tools to implement rules shown in Figure 1 are provided by the fuzzy set theory, known as "logic primitives" [3]. Boolean operators like AND, NOT, OR, (normal sets i.e. t-norm and t-conorm sets respectively) produce a crisp result if the universal discourse provided is a crisp by itself. These same operators are used to formulate the implied functions giving a choice of a finite set of logic primitives that are obtained as a subset of the logical operations performed on a particular universal discourse. The ultimate goal of fuzzy logic is to provide mathematical

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foundation for approximate reasoning when imprecise proposition are expressed in natural langage with the help of linguistic variables.

Fuzzy set union and intersection classes:
A fuzzy operation, as compared to a crisp operation, is performed on two operators, which itself is a region between 0 and 1, instead of any particular value and again the output of the operation is a region between 0 and 1.Morever a fuzzy function must satisfy the following properties in order to be fuzzy union and intersection.

- Commutation $S(a,b) = S(b,a)$
- Association $S(a,S(b,c)) = S(S(a,b),c)$
- Monotonic $S(a,b)\ \Omega\ S(c,d)$ *if* $a\ \Omega\ c$ *and* $b\ \Omega\ d$
- One Identity for norm $S(a,0) = a$
- Zero Identity for conorm. $S(a,1) = 1$

Zadeh in 1965 defined the very basic formula for Union and Intersection [6], but are still considered as standard. One active research area is the development of parameterized families of intersection and union. Table 1 consists some of the famous families for intersection and union.

Generalization of implication normal forms
To start with, we use the very basic functions Min and Max. They are proposed as intersection and union operators for fuzzy sets [1,3,4,5,6]. In the second step we execute the implication function, based on these operators, extending its classical definition. This leads us to two classes of implications based on the two canonical forms of $p \rightarrow q$ namely

- the conjunctive normal form $\neg\mu(p) \wedge \mu(q)$ .....(1)
- the disjunctive normal form $\neg\mu(p) \vee (\mu(p) \wedge \mu(q))$ .....(2)

Where $\mu(p)$, $\mu(q)$ → truth values of propositions p and q respectively.

Another fuzzy operator is generated by a mathematical concept, known as *quasi inverse of a continuous t-norm AND* [3]. It is specified as

$$\mathbf{IMPLY}(\mu p , \mu q) = \mathbf{Max}[\mu(k)\ \varepsilon[0,1] \mid \mathbf{AND}\ (\ \mu(k), \mu(p)\ )\ \Omega\ \mu(q)\ ]^1 \quad \text{.....(3)}$$

This class 3 of fuzzy implication is generated by a more complex way. Looking at equation(3), we have three values p, q, and k. Here it should be noted that k is an arbitrary parameter which is used only to provide a relation between p and q. To start with, if we see at the crisp values for p, q and k, we have :

K	P	Q	P → Q	P AND K	V(P AND k) <= v(Q)	V(K) <= v(p→ q)
0	0	0	1	0	1	1
0	0	1	1	0	1	1
0	1	0	0	0	1	1
0	1	1	1	0	1	1
1	0	0	1	0	1	1
1	0	1	1	0	1	1
1	1	0	0	1	0	0
1	1	1	1	1	1	1

Table 1: Crisp Space for p, q & k and their equivalence relations.

In order to qualify as a fuzzy measure, a function f(x) must have certain properties. These required properties are based on the following axioms:

Axiom 1: *(boundary conditions)*
$f(\phi) = 0$ and $f(X) = 1$

Axiom 2: *(monotonicity)*
For every $p, q \varepsilon P(X)$, if $p \subseteq q$, then $f(p) <= f(q)$

Axiom 3: *(continuity)*
For every sequence $(P_i \varepsilon P(X) \mid i \varepsilon N)$ of subsets of X, either $P_1 \subseteq P_2 \subseteq P_3 \ldots$ or $P_1 \supseteq P_2 \supseteq P_3 \ldots$ (i.e. the sequence is monotonic) [6].

If we look at the columns 6 and 7 of table 1, we see an equivalence relation that the crisp set follows for implication. Hence we can say that the equation derived from the quasi inverse operator holds true for the crisp logic values, i.e. the boundary values , proving axiom 1.

By elementary algebra, we have :

$k <= p * q \leftrightarrow k/p <= q$

The problem is to generalise the equivalence relation to infinite - valued logic [0,1]. Here the argument is if $p = \mu(P)$ and $q = \mu(Q)$, then do we have an operator T*(p,q) exist which should be the inverse of AND (p,q) i.e. which should say,

$AND(p,k) <= q \leftrightarrow k <= T^*(p,k)$..... **(4)**

Since , AND in fuzzy can be replaced by

AND (a,b) = Min [a,b] ... *from Zadeh family*

So we have

$T^*(p,q)=\max(k\varepsilon[0,1] \mid AND(k,p)) <= q$,.. **(5)**

Where T* is unique if AND is continuous [3]. Therefore T* can be interpreted as a truth function of an implication. Hence the formula given in equation (3) holds true for fuzzy implication, proving axiom 2 and axiom 3 stated above.

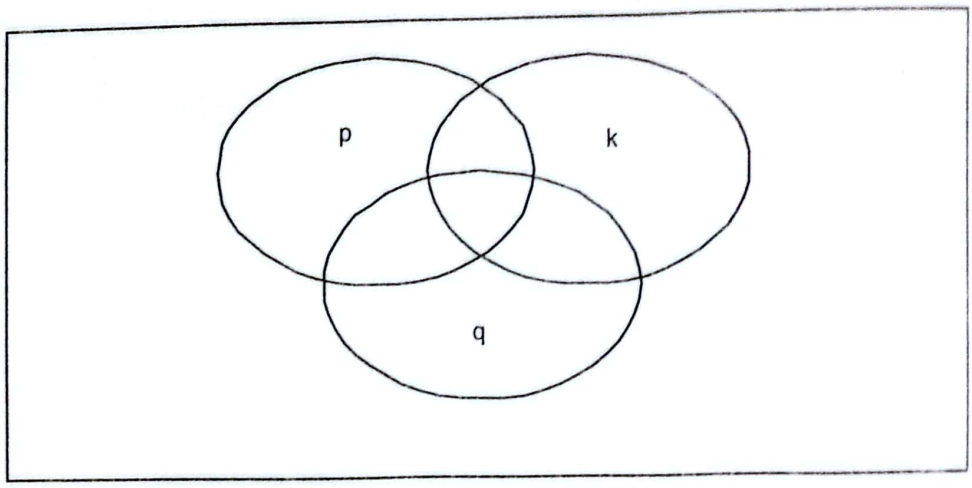


Fig 2. Venn Diagram for p, q and k

The parameter 'k' plays a very important role. If k is joint with p and q, then the implication formula for first two classes state perfectly well, i.e. OR(NOT(p),q). In other words q is the implied result.

If some part of k is common to p and q then the implied result is that area where p and k intersect and should be less than q. The shaded area in figure 2 is the implied result, if quasi inverse technique is chosen.

Examples for Class 3 Implication:

Following are the examples of quasi inverse fuzzy implication operation:

$$Imply(x, y) = \begin{cases} 1 \text{ if } x <= y \\ y \text{ otherwise} \end{cases} \quad \ldots\ldots\ldots (6)$$

$$Imply(x,y) = \begin{cases} Min(y/x, 1) \text{ if } x \neq 0 \\ 1 \text{ otherwise} \end{cases} \quad \ldots\ldots\ldots (7)$$

Equation (6) is known as BROWER GODEL implication and equation (7) is known as GOGUEN implication.

Since class 3 implication is given as,

IMPLY[p,q] = Max[k| And(p,k) <= q]

If AND (k, p) is taken as Max[p , k] , then at the following boundary values

$p=1$ i.e. $k<= q \rightarrow q$ i.e. $T(1, q) = q$
$k=1$ i.e. $p<= q \rightarrow 1$ i.e. $T(p, q) = 1$ where $p<= q$
$max[0] = 0 <= q \rightarrow 0$ i.e. $T(0, q) = 0$

which proves Equation (6).

If AND(k,p) is taken as p*q (simple multiplication) , then we have,

IMPLY[p,q] = Max[k| And(p,k) <= q]

= Max [k| p * k <= q]
= Max [k| k / p >= q]
p = 1 → p<=q → $T(1, q) = q$
k = 1 → p<=q → $T(p, q) = 1$
(k/p) <= q → $T(p,q) = 1$ where $p<1$

The above derivation proves Equation (7), where Min[q/p,1] is taken to make sure that the final value should remain in the region of [0,1].

Practical Implementation for Class 3 implication :

We used Mathematica to plot the Brower Godel and Goguen implementations. The syntax for Gougen is given as,

Imply Gougen [p_,q_]:=Which[p!=0,Min[q/p,1],p==0,1]
Plot3D[implyGougen[p,q],{p,0,1},{q,0,1}]

The surface diagram for Gougen comes as,

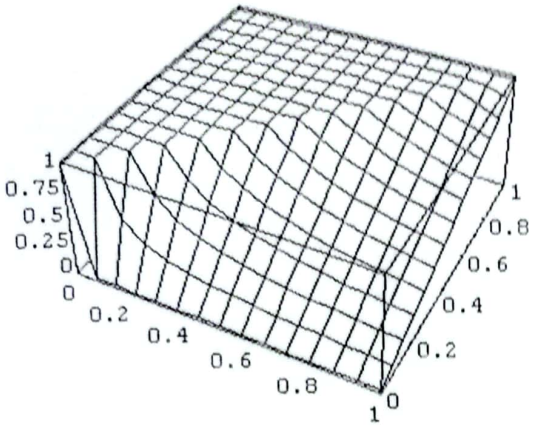


Figure 3:Gougen Implication Surface Area

The Mathematica code for Brower Godel is given as,

implyBrower[p_,q_]:=Which[p<=q,1,p>q,q]
Plot 3D [implyBrower[p,q],{p,0,1},{q,0,1}]

And the surface area comes out to be,

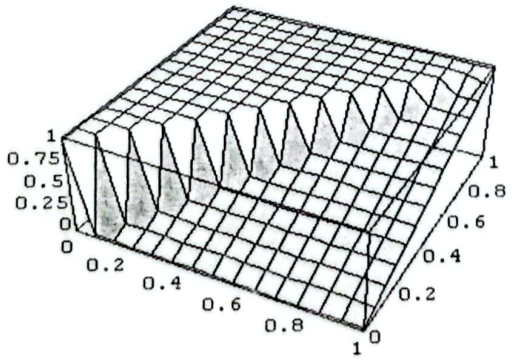


Figure 4: Brower Godel implication

By looking at Figure 4 and Figure 5, it is clear that the quasi inverse operator works perfectly well for implementation. Another class 3 operator, which is not so accurate as Gogeun or Godel is Rescher - Gaines implication. The mathematica code for Rescher - Gaines is given as,

implyRescher[p_,q_]:=Which[p<=q,1,p>q,0]
Plot 3D [implyRescher[p,q],{p,0,1},{q,0,1}]

The graph for Recher-Gaines implication operator is given in Figure 5.

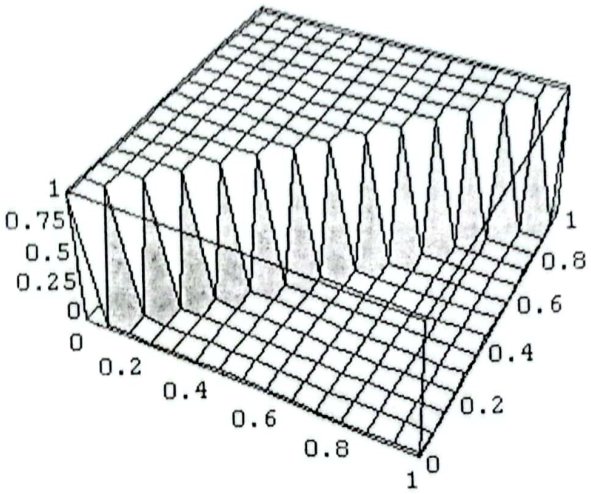


Figure 5 Recher - Gaines Implication

The Recher - Gaines implication operator works on a threshold value of 0 and 1, i.e. if p is less than q, the result is 1 and 0 otherwise. Therefore this implication family is not very commonly used. However in control systems, where the calculation time required is very less or the output is required in Real Time, this formula is used.

Results & Practical demonstration of Class 1 and Class 2 Implications

Apart from class 1 and class 2 formulae, given in table 1, following are some more implication formulae, used commonly. Figures 6 - 9 show thier surface graphs.

Author	Implication	Type
Reichenbach	1 - p + pq	Class 1
Willmott	Max(1-p, Min (p,q))	Class 2
Kleive Dienes	Max(1-p, y)	Class 2
Lukasiewicz	Min(1-p+y, 1)	Class 2

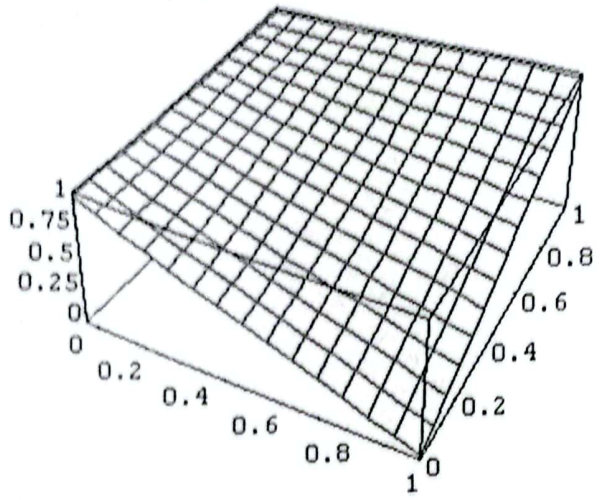


Figure 6 Reichenbach implication plot

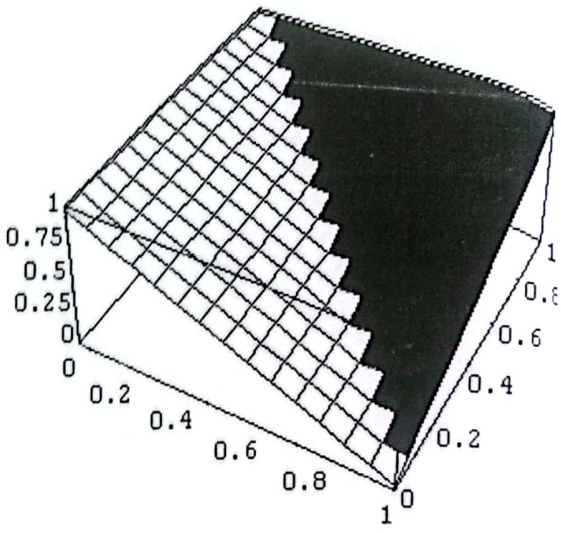


Figure 7 Kleive Dienes implicaton plot

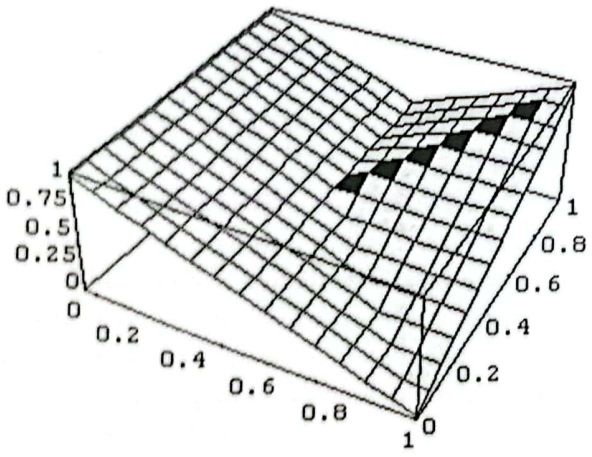


Figure 8 Willmott implicaton

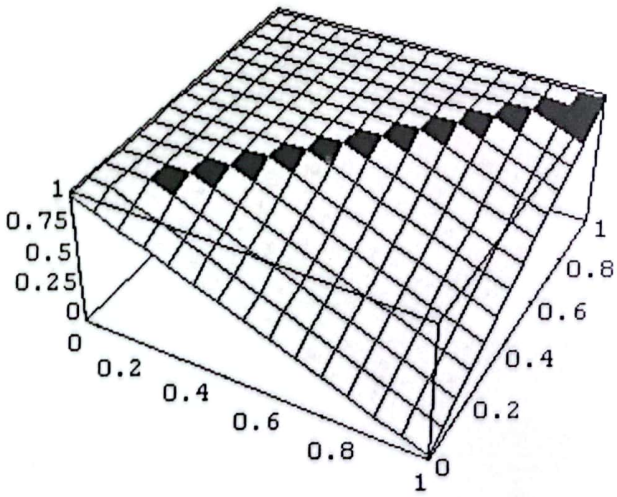


Figure 9 Lukasiewicz implicaton

The above four formulae certainly fit for fuzzy implication, because when we see the figures above, we find that all these equations obey the crisp value sets, their values are monotonic and are in continuity. Hence the above equations can be used in any type of fuzzy domain problem.

Summary of the Results:

1. The operator " quasi inverse of a continuous t-norm AND", called class 3 implication, qualifies as a fuzzy measure.
2. The operator " quasi inverse of a continuous t-norm AND", called class 3 implication, qualifies as a fuzzy measure.
3. B. Godel and Goguen implication are class 3 implications.

4. Surface graphs of same famous implications like Willmott, Kleive Dieness, etc here shown that they fit for fuzzy implications as they obey the crisp value sets, their values being monotonic and in continuity.

Conclusion

Fuzzy system using implications can be effectively used in different areas such as process control, modelling, decision making, analysis, etc.

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