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PRIMARY IDEALS IN BRANCH ALGEBRA OF ANALYTIC FUNCTIONS

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ABSTRACT

Let A(D) be a Branch Algebra of Analytic Functions on a subset D of the complex plane. Thus if A(D) is a Noetherian Ring, b and d are ideals in A(D), where b is the radical of d. It is shown that if $b^{ll} < d$ for some integer then d is a primary ideal in A(D).

Key words: **Primary ideals, analytic functions**

Introduction

Henriksen [5], Muhly [6], Helon and Quigly [7] Cohen [8], Schark [9], Arens [10]] and Gillman [11] have determined maximal ideals in certain rings of analytic functions and continuous functions.

This paper contains the characterization of closed primary ideals in Noetherian rings in Banach algebra of Analytic functions.

Preliminaries:

<u>Definition 1.1:</u> let D be a region in the complex plane and A(D) be the family of all Analytic function with ascending chain condition. With usual definitions of addition, multiplication, scalar multiplication, supremum norm and uniform limit of function in A(D), the Noetherian ring becomes a Banach algebra "or so to say" a topological ring of analytic functions on D.

<u>Definition 1.2</u>: (Maximal principal ideals and differential idelas) Following [2], [3] and [4], every element of A(D) generates a principal ideal (f) that is, the set of all elements of the form fh, hεA(D). (f) is said to be a maximal principal ideal if $(f) \neq A(D)$ and if $(f) \subset (g) \neq A(D)$ implies that (f)=(g).

Moreover, (f) is a maximal principal ideal if any only if (f)=I=[f:fε A(D), f(α)=0] for every α ε D.

An ideal I in A(D), [4: P.115] is called a differential ideal if for any vector space E, f'ε E whenever fε I where E is a vector space of analytic function over D.

<u>Definition 1.3</u> A primary ideal d in A(D) is contained in a unique maximal ideal. Let b and d be ideals in A(D) such that $b^{ii} \subset d$ and b is a radical of d for some integer n.

<u>Definition 1.4:</u> Since an ideal I and A(D) is proper and does not contain 1 which implies that I consists of non-invertible elements of A(D) and the set non-invertible element being a closed ideal. In particular, every proper ideal in A(D) is a closed ideal. This implies the primary ideal d is closed.

(Finitely General Idealas)

<u>Definition 2.1:</u> O. Helmer [1: TH.9, p.351] has proved the following theorem:

<u>Theorem 2.2:</u> In A(D) every finitely generated ideal is a principal ideal.

<u>Theorem 2.3:</u> In A(D) every principal ideal is a closed differential ideal. Proof is abovious.

Residue Class Rings:

<u>Definition 3.1:</u> Since A(D) is a commutative ring with identity element and I is an ideal in A(D), A(D)/I denotes the following ring:

For f, gε A(D), define f≡g (mod I) to mean that f-gεI. Clearly ≡ is an equivalence relation and this is compatible with the operations in A(D).

This implies if f≡ g, h≡k then f+h≡g+k and

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fh≡gk. The equivalence relation ≡ partitions A(D)/I, into disjoint equivalence classes. Let us denote by [f] the equivalence class that contains f.

Defining [f]+[g] = [f+g], [f] [g] = [fg], we have the quotient ring A(D)/I to be called a residue class ring.

For any given natural homomorphism Φ: onto

A(D) ---------> A(D)/I, Φ(f) =[f] which satisfies Φ^{-1} (0) = I.

Theorem 3.2: Let I be a maximal ideal in the Banach algebra A(D). The I is primary ideal.

Since A(D) is a commutative ring with identity element and I is an ideal is A(D), A(D)/I denotes the following ring:

Fro f,gεA(D), define f≡ g, h≡ k then f+ h≡g + k and fh≡gk. The equivalence relation ≡ partitions A, into disjoint equivalenece classes let us denote by [f] the equivalence class that contains f.

Defining [f]+[g]=[f+g], [f] [g] = [fg] = [fg] we have the quotient ring A(D) / I to be called a residue class ring. For any given natural homomorphism Φ: A(D) onto A(D)/I, Φ(f) = [f] which satisfies Φ^{-1} (0)=I.

The closed primary ideals in A(D) are characterized by the following theorem:

Theorem 3.3: Let I be a maximal ideal in the Banach algebra A(D) then I is primary ideal.

Proof: Since I is a maximal ideal in A(D), A(D)/I is a field. But A(D)/I is a residue class ring by definition 3.1 which means for f,gε A(D), there is a natural homomorphism Φ: A(D) -----(A(D)/I such that Φ (f)=[f] with Φ^{-1}(0) = i. Since A(D)/I is a field, the equation [f] [h] = [1] has a unique solution for [f] whenever [f], [h]ε A(D)/I and [h] ≠ 0.

This means we must have fε A(D) which satisfies fh=1 εI whenever hεI. Since any two such f_s not necessarily be both elements of A(D), must differ by an element of I=> (f'-f) hε I=> f' h-λεI and fh-λεI. Taking h=f in fh-λ εI, we obtain ff'-λ εI, λ εI, λ εI. Since every maximal ideal is a principal ideal generated by a single element, the ideal I is generated by f, and f, f'ε A(D), ff'ε d, f' ε d implies there exists a positive integer n such that f^{11} εd, b^{ii} ⊂ d for an ideal b in A(D) thereby implying that d is a primary ideal in A(D).

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