# AUTOMATED ANALYSIS OF PUMPING TESTS IN CONFINED NON LEAKY AQUIFERS

Shabbir A.S. Sayed* and Ali Nawaz Shah Hussaini**

**Abstract**

The paper describes time-drawdown equations for confined non leaky aquifers, pumped by partially or fully penetrating wells and discusses the method of fitting a least square curve to the field data. A program in IBM BASIC is presented which determines directly the aquifer parameters. The program is quick and has been tested using data available in literature. Automated pumping test analysis has the advantage of objectivity which is lacking in graphical methods. The results are close to those obtained by graphical methods.

## Intorduction

With easy availability of desktop computers and hand calculators, automated analysis of pumping tests is becoming increasingly popular, In automated analysis, aquifer parameters are determined by fitting a least square curve to the field data. The method has the advantage of automatic objectivity which is lacking in the type curve matching or other graphic methods.

In a previous paper (Sayed 1982), two hand calculator programs were given for pumping test analyses by least squares method. The modified Theis equation (Cooper and Jacob 1946) was programmed for time-drawndown and distance-drawdown. In this paper the procedure of automated analysis of data resulting from pumping a fully confined, isotropic, nonleaky, aerealy infinite, fully or partially penetrated aquifer has been outlined. The computer program used to carry out analysis is also listed. The program calculates partial penetration factor (Hantush 1964) and computes directly transmissivity and storativity by least squares method. The program is written in BASIC for IBM P.C/XT or compatibles.

* Institute of Irrigation and Drainage Engineering, Mehran University of Engineering and Technology, Jamshoro, Sindh, Pakistan.
** Assistant Professor, Department of Statistics, University of Sindh, Jamshoro.

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## Partial Penetration Equation

The drawdown in a piezometer, having a depth of penetration z and being at a distance " r " from a steadily discharging well, that is screened between depths 'd' and 'l' in an isotropic, non-leaky, fully confined aquifer of thickness b (Fig 1), is given by Hantush (1964)

$$s = \frac{Q}{4\pi T}\left[W(u) + F(u, r/b, d/b, l/b, z/b)\right] \quad (1)$$

Similarly the drawdown in an observation well screened between 'd' and 'l' (Fig 1) is given by Hantush (1964).

$$s = \frac{Q}{4\pi T}\left[W(u) + \bar{F}(u, r/b, l/b, d/b, l'/b, d'/b)\right] \quad (2)$$

where

W (u)= well function for non leaky confined fully panetrated aquifers.

$$= -0.5772 - \text{Ln } u - \sum_{n=1}^{\infty} \frac{(-1)^n u^n}{n.n!} \quad (3)$$

$$u = r^2 S/4Tt \quad (4)$$

F = partial penetration factor for piezometer

$$= \frac{2b}{(l-d)} \sum_{n=1}^{\infty} \frac{1}{n}\left(\sin\frac{n\pi 1}{b} \sin\frac{n\pi d}{b}\right)$$

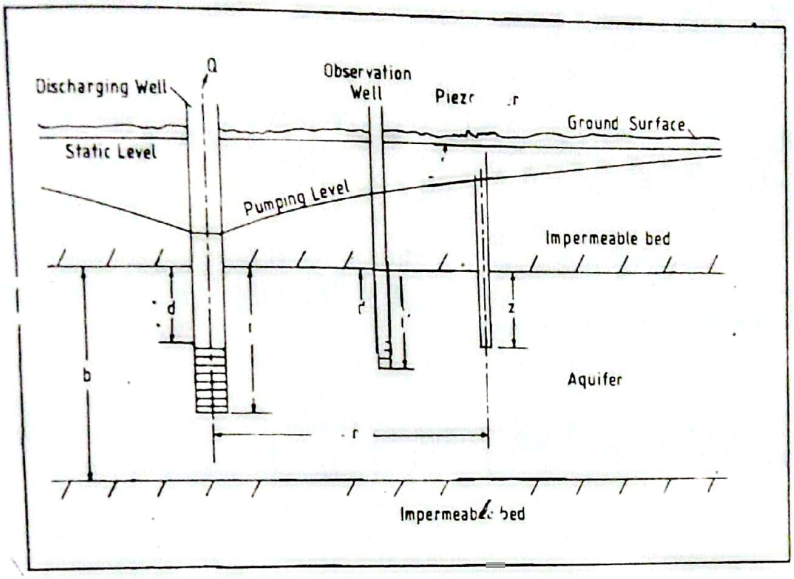
$$\cos\left(\frac{n\pi s}{b}\right) W\left(u, \frac{n\pi r}{b}\right) \quad (5)$$

$\bar{F}$ = partial penetration factor for observation well

$$= \frac{2\,b^2}{\pi^2 (l-d)\,(l'-d')} \sum_{n=1}^{\infty} \frac{1}{n^2}\left(\sin\frac{n\pi l}{b} - \sin\frac{n\pi d}{b}\right)$$

$$\left(\sin\frac{n\pi l'}{b} - \sin\frac{n\pi d'}{b}\right), W\left(u, \frac{n\pi r}{b}\right) \quad (6)$$

W (u,) is well function for leaky aquifers, with leakance = 0 (nonleaky aquifers).




where

b = aquifer thickness (L)

d = distance from top of aquifer to the top of screen of discharging well (L);

d' = distance from top of aquifer to the top of observation well screen (L);

l = distance from the top of aquifer to the bottom of screen of discharging well (L).

l' = distance from top of aquifer to the bottom of observation well screen (L);

Q = discharge rate $\left(\frac{L^3}{T}\right)$;

r = radial distance from the centre of discharging well (L);

S = storativity (dimensionless);

T = transmissivity $\left(\frac{L^2}{T}\right)$;

t = time since pumping started (T);

z = depth of penetration of the peizometer measured from the top of aquifer (L);

The angles in the series are in radians.

For relatively large time W (u,B) can, for all practical purposes, be replaced by $2K_O\left(\frac{n\pi r}{b}\right)$. The series in Eqs (5) and (6), thus become independent of time and Eqs (1) and (2) become:

$$s = \frac{Q}{4\pi T}\left[W(u) + f\right] \tag{7}$$

$$s = \frac{Q}{4\pi T}\left[W(u) + \bar{f}\right] \tag{8}$$

$$f = \frac{4b}{(l-d)} \sum_{n=1}^{\infty} \frac{1}{n}\left(\sin\frac{n\pi l}{b} - \sin\frac{n\pi d}{b}\right)\mathrm{Cos}\left(\frac{n\pi z}{b}\right) \cdot K_o\left(\frac{n\pi r}{b}\right) \tag{9}$$

$$\bar{f} = \frac{4b^2}{\pi^2 (l-d)(l'-d)} \sum_{n=1}^{\infty} \frac{1}{n^2}\left(\sin\frac{n\pi 1}{b} - \sin\frac{n\pi d}{b}\right)$$

$$\left(\sin\frac{n\pi l'}{b} - \sin\frac{n\pi d'}{b}\right) \cdot K_o\left(\frac{n\pi r}{b}\right) \tag{10}$$

where

$K_0(x)$ = Zero order modified Bessel function of second kind

$$K_0(x) = \mathrm{Ln}\frac{1.12}{x} \quad \text{for } x \leq 0.5$$

$$K_0(x) = \sqrt{\frac{\pi}{2x}}\left(1 - \frac{1}{8x}\right) e^{-x} \quad \text{for } x \geq 0.5 \text{ (Warner and Yow 1980)}$$

For small values of u ( $u \leq 0.01$), the terms in the series in Eq (3) beyond Ln u become insignificant (Copper and Jacob 1947) and Eq (b) and Eq (7) are reduced to

$$s = \frac{Q}{4\pi T}\left(-0.5772 - \mathrm{Ln}\, u + F\right) \tag{11}$$

where F' stand for f or $\bar{f}$ as the case may be.

$$\text{or} \quad s = \frac{Q}{4\pi T}\left(\mathrm{Ln}\frac{2.25\,Tt}{r^2 S} + F'\right) \tag{12}$$

$$\text{or} \quad s = A + B\,\mathrm{Ln}\, t \tag{13}$$

$$\text{where} \quad A = \frac{Q}{4\pi T}\left(\mathrm{Ln}\frac{2.25\,Tt}{r^2 S} + F'\right) \tag{14}$$

$$B = \frac{Q}{4\pi T} \tag{15}$$

Rearranging Eqs (14) and (15)

$$T = \frac{Q}{4\pi B} \tag{16}$$

and

$$S = \frac{2.25\ T\ e^{-}\ (4\pi TA/Q - F')}{r^2} \tag{17}$$

The coefficients A and B can be calculated by least squares method by the following formulae (Speigel 1972)

$$A = \frac{1}{N}\ (\sum Y - B \sum X) \tag{18}$$

$$B = \frac{\sum XY - \bar{x} \sum Y}{\sum x^2 - \bar{x} \sum x} \tag{19}$$

where X denotes Lnt and Y denotes s. N is the number of data sets. In order to know how well the data fits the curve, the "coefficient of determination' may also be calculated. It is a dimensionless quantity ranging between 0 and 1. The best fit is achieved when its value is closer to unity. The coefficient of determination has the following equation (Speigel 1972).

$$d^2 = \frac{A \sum Y + B \sum XY - (\sum Y)^2/N}{\sum Y^2 - (\sum Y)^2/N} \tag{20}$$

The partial penetration factor F' may be calculated from Eq (7) or Eq (8) for a few terms of the series. Larger values of n do not make any significant change in the value of F'.

For a fully penetrating well, l = b and d = 0, the series in the Eq (7) and (8) vanish and F' = 0. The equations therefore, are equivalent to Theis formula for fully penetrating well.

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## The program

The program is written in Advanced BASIC of IBM and can be used on IBM P.C. series or their compatibles. It is expected that the user of the program is familiar with BASIC programing language. The program is listed in Appendix A. The mathematical symbols and their equivalents in the program are given in Table 1.

The program can handle full and partial penetration conditions. The time-drawdown data may either be from a piezometer or an observation well. The various steps of the program are documented in Table 2. The program has been tested using data for a fully penetrating well (Todd 1969, p. 92) and and a partially penetrating well (Walton 1970 p. 285). Sample outputs are given in Appendices B and C.

The following comparision will show that the results of the automated analysis are very close to those obtained by graphical methods.

(1) Full penetration condition:

(a) Program results: $T = 0.157\ Ft^2/s$ $S = 1.80 \times 10^{-4}$

(b) Graphical Method $T = 0.159\ Ft^2/s$ $S = 1.98 \times 10^{-4}$

(2) Partial penetration conditions:

(a) Program results $T = 0.415\ Ft^2/s$ $S = 3.74 \times 10^{-4}$

(b) Graphical method $T = 0.369\ Ft^2/s$ $S = 3.2 \times 10^{-4}$

## Conclusion

The objectivity and speed makes automated pumping test analysis very attractive compared with the traditional graphical methods. The program can be used, with confidence, if the field conditions conform to the theoritical boundary conditions.

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## References


Cooper, H.H and C.E. Jacob., 1946. A generalised graphical method for evaluating transmissibility and storage coefficient from pumping test. Am. Geophys. Union Trans., **27**, pp. 202-214.

Hantush, M.S., 1961. Drawdown around a partially penetrating well. Hydraulic Div. Jour. Am. Soc. Civil Engineers. Proc., pp 83-98.

Hantush, M.S., 1964. "Hydraulics of Wells" IN Advances in Hydrosciences V.1, V.T. Chow, ed. Academic Press, New York, pp 281-432.

Kayed, S.A.S., (1982) Programmable hand calculator programs for pumping test analyses by least squares method using Jacob's modification of Theis Equation. Ground Water **20**, No. 1. 1982.

Spiegel M.R., (1972) Statistics. Schum's Series, McGraw Hill, pp 242-243.

Todd, D.K., (1959) Ground Water Hydrology, John Wiley.

Walton W.C., (1970) Ground Water Resource Evaluation. McGraw Hill.

Warner, D.L and M.G. Yow., (1980) Programmable hand calculator program for pumping and injection wells-III-Constant pumping (injection) rate, fully confined aquifer, partially penetrating well. Ground Water, **18**, No.5, pp. 438-443.


**Table 1.** Mathematical symbols in various equations and their equivalents in the program.

| Mathematical Symbol | BASIC Equivalent |
|---|---|
| A | A1 |
| B | B1 |
| b | B |
| d | D |
| d' | DPRIME |
| f, $\bar{f}$ | F |
| $K_0$ (x) | K0 (A) |
| l | L |
| l' | LPRIME |
| N | K |
| n | N |
| Q | Q |
| r | R |
| S | STOR |
| $s_i$ | DR (I) |
| T | TRANS |
| $t_i$ | T(I) |
| z | Z |

**Table 2. Program documentation.**

| *Line No.* | *Action.* |
|---|---|
| 10-20 | Program heading. |
| 30 | Provides storage for time-drawdown data. |
| 35 | Read P and PP. P=1 indicates piezometer and P=2 indicates observation well. PP=1 and PP=2 indicate full and partial penetration conditions respectively |

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|---|---|
| 40 | Reads observation well dimensions and discharge. |
| 45-170 | Print observation well and piezometer or observation well dimensions |
| 180-230 | Initialize summation as in Equation (5) and (6). |
| 250-350 | Reads time-drawdown data and summation is performed as in equation (5) and (6) till the program encounters a time value. |
| 360-370 | Calculate X |
| 380-395 | Calculate B, A d2 |
| 400 | Checks for full or partial penetration condition. |
| 410-510 | Calculate f, T |
| 520-530 | Calculate T and S |
| 540-630 | Print results. |

**Appendix A. Listing of BASIC program for analysis of pumping test data.**

```
10    REM program for automated pumping test analysis for confined, nonleaky fully
20    REM or partially penetrated aquifer... by Shabbir.A.S.Sayed.
30    DIM T (40), DR (40)
35    INPUT P.PP
40    INPUT B,D,L,Q
45    LPRINT TAB (5) "PUMPING TEST ANALYSIS--";
50    IF PP=1 THEN LPRINT "FULL PENETRATION CONDITIONS-JACOB METHOD" ELSE
      LPRINT" "PARTIAL PENETRATION CONDITIONS -HANTUSH METHOD"
55    LPRINT USING "AQUIFER THICKNESS= ###.##; B;:LPRINT "Feet"
60    LPRINT USING "DEPTH TO TOP OF ABST.WELL SCREEN=###.##";L;:LPRINT "Feet"
70    LPRINT USING"DEPTH TO BOTTOM OF ABST.WELL SCREE= ###.##";L;:LPRINT "Feet"
80    LPRINT USING"WELL DISCHARGE= ##.#";Q"LPRINT "CUSECS"
100   IF P=1 THEN INPUT R,Z
110   IF P=2 THEN INPUT R,DPRIME,LPRIME
120   REM P=1 indicates piezometer and P=2 indicates observation well.
130   IF P=1 THEN LPRINT USING "DISTANCE TO PIEZOMETER= ###.##";R;: LPRINT "Feet"
140   IF P=1 THEN LPRINT USING"DEPTH OF PIEZOMETER= ###.##";Z;: LPRINT "Feet"
150   IF P=2 THEN LPRINT USING"DISTANCE TO OBSERVATION WELL= ###.##";R;:LPRINT "Feet"
160   IF P=2 THEN LPRINT USING"DEPTH TO TOP OF OBS. WELL SCREEN=
      ###.##";DPRIME;:LPRINT "Feet"
170   IF P=2 THEN LPRINT USING"DEPTH TO BOTTOM OF OBS.WELL SCREEN=
      ###.##";LPRIME;:LPRINT "Feet"
180   SUMX=0
190   SUMY=0
200   SUMXY=0
210   SUMX2=0
220   SUMY2=0
230   SUMF=0
240   PI=3.141593
250   I=1
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```
260   INPUT T(I),DR(I):IF T(I)<0 THEN GOTO 360
270   REM DR(I) is drawdown at time T(I)
280   X=LOG(T(I)*60):REM convert minutes to seconds.
290   Y=DR(I)
300   SUMX=SUMX+X
310   SUMY=SUMY+Y
320   SUMXY=SUMXY+X*Y
330   SUMX2=SUMX2+X*X
340   SUMY2=SUMY2+Y*Y
345   I=I+1
350   GOTO 260
360   K=I-1
370   XAV=SUMX/K
380   B1=(SUMXY-XAV*SUMY)/(SUMX2-XAV*SUMX)
390   A1=(SUMY-B1*SUMX)/K
395   D2=(A1*SUMY+B1*SUMXY-SUMY*SUMY/K)/(SUMY2-SUMY*SUMY/K)
400   IF PP=1 GOTO 520:REM PP=1 indicates full penetration conditions.
405   FOR N=1 TO 10
410   NPIB=N*I/B
415   NPILP=NPIB*LPRIME
420   NPIDP=NPIB*DPRIME
425   NPIL=NPIB*L
430   NPID=NPIB*D
435   NPIZ=NPIB*Z
440   NPIR=NPIB*R
450   IF NPIR<=.5 THEN DEF FNK0(A)=LOG(1.12/A)
460   IF NPIR>.5 THEN DEF FNK0(A)=SQR(PI/2*A)*((1-1/(8*A))*EXP(-A)
465   REM K0(A) is zero order modified Bessel function of second kind.
470   if p=1 then somf-somf+(sin(npil)- SIN(NPID))*COS(NPIZ)*COS(NPIZ)*FNK0(NPIR)/N
480   IF P=2 THEN SUMF=SUMF+(SIN(NPIL)- SIN(NPID))*(SIN(NPIDP))*FNK0(NPIR)/(N*N)
490   NEXT N
500   IF P=1 THEN F=4*B*SUMF/(PI*(L-D))
510   IF P=2 THEN F=4*B*B*SUMF/(PI*PI*(L-D_*(LPRIME-DRPIME))
520   TRANS=Q/ (4*PI*B1)
530   IF PP=1 THEN ARG=4*PI*TRANS*A1/Q ELSE ARG=4*PITRANS*A1/Q-F
540   STOR=2.25*TRANS*EXP)-ARG)/(R*R)
550   LPRINT TAB(3) "TIME" TAB(10) "DRAWDOWN"
560   LPRINT TABL(3) "Feet" TAB(12)"Feet":LPRINT
570   FOR I=1 TO K
580   LPRINT USING" ####.##":T(I),DR(I)
585   NEXT I
```

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```
590     LPRINT
600     LPRINT USING"TRANSMISSIVITY=##.##^^^^";TRANS;: LPRINT "FT^2/S"
610     LPRINT USING"STORATIVITY=##.##^^^^";STOR
620     LPRINT USING"PARTIAL PENETRATION FACTOR=##.###";F
630     LPRINT USING"COEFFICIENT OF DETERMINATION = ##.###";D2
640     END
```

**Appendix B. Program output - Full penetration conditions.**

PUMPING TEST ANALYSIS--PARTIAL PENETRATION CONDITIONS -HANTUSH METHOD

AQUIFER THICKNESS= 600.00 Feet

DEPTH TO TOP OF ABST.WELL SCREE=450.00 Feet

DEPTH TO BOTTOM OF ABST.WELL SCREEN=450.00 Feet

WELL DISCHARGE= 2.3 CUSECS

DISTANCE TO OBSERVATION WELL= 300.00 Feet

DEPTH TO TOP OF OBS. WELL SCREEN=450.00 Feet

DEPTH TO BOTTOM OF OBS.WELL SCREEN= 600.00 Feet

| TIME Feet | DRAWDOWN Feet |
|---|---|
| 1.00 | 0.69 |
| 2.00 | 0.99 |
| 4.00 | 1.16 |
| 4.00 | 1.30 |
| 5.00 | 1.40 |
| 6.00 | 1.50 |
| 7.00 | 1.55 |
| 8.00 | 1.62 |
| 9.00 | 1.68 |
| 10.00 | 1.74 |
| 20.00 | 2.10 |
| 30.00 | 2.26 |
| 40.00 | 2.46 |
| 50.00 | 2.51 |
| 60.00 | 2.58 |
| 70.00 | 2.65 |
| 80.00 | 2.70 |
| 90.00 | 2.76 |
| 100.00 | 2.80 |
| 200.00 | 3.12 |
| 300.00 | 3.28 |
| 400.00 | 3.35 |

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|---|---|
| 500.00 | |
| 600.00 | 3.42 |
| 700.00 | 3.48 |
| 800.00 | 3.55 |
| 900.00 | 3.66 |
| 1000.00 | 3.70 |
| 1440.00 | 3.70 |
| | 3.90 |

**Appendix C. Program output - Partial penetration conditions.**

TRANSMISSIVITY= 4.15E-01 FT^2/S
STORATIVITY= 3.74E—04
PARTIAL PENETRATION FACTOR= 1.133
COEFFICIENT OF DETERMINATION = 0.997

PUMPING TEST ANALYSIS--FULL PENETRATION CONDITIONS-JACOB METHOD
AQUIFER THICKNESS=500.00 Feet
DEPTH TO TOP OF ABST.WELL SCREEN= 0.00 Feet
DEPTH TO BOTTOM OF ABST.WELL SCREEN=500.00 Feet
WELL DISCHARGE= 1.1 CUSECS
DISTANCE TO PIEZOMETER= 200.00 Feet
DEPTH OF PIEZOMETER= 500.00 Feet

| TIME | DRAWDOWN |
|---|---|
| Feet | Feet |
| 1.00 | 0.66 |
| 1.50 | 0.87 |
| 2.00 | 0.99 |
| 2.50 | 1.11 |
| 3.00 | 1.21 |
| 4.00 | 1.36 |
| 5.00 | 1.49 |
| 6.00 | 1.59 |
| 8.00 | 1.75 |
| 10.00 | 1.86 |
| 12.00 | 1.97 |
| 14.00 | 2.08 |
| 18.00 | 2.20 |
| 24.00 | 2.36 |
| 30.00 | 2.49 |
| 40.00 | 2.65 |
| 50.00 | 2.78 |
| 60.00 | 2.88 |

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| | |
|---|---|
| 80.00 | 3.04 |
| 100.00 | 3.16 |
| 120.00 | 3.28 |
| 150.00 | 3.48 |
| 180.00 | 3.51 |
| 240.00 | 3.67 |

TRANSMISSIIVITY= 1.57E-01 FT^2\S

STORATIVITY= 1.80E—04

PARTIAL PENETRATION FACTOR= 0.000

COEFFICIENT OF DETERMINATION = 0.999

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