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On the Development of Numerical Iterated Method of Newton Raphson method for Estimating Nonlinear Equations

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Abstract: This paper used the Newton-type Method for estimating a single root of nonlinear equations. This method is iterative method and also known as one of the open methods. Open method are fast converging method as compared to closed method but the convergence is not guaranteed. Since open methods are fast convergence methods that is why they are widely used in Applied Mathematics. The proposed numerical technique is second order of convergence, and which is based on Newton Raphson method. The developed algorithm is compared with the well-known Newton Raphson Method and results show that our developed method is much better than the well-known method. Furthermore, examples are also give in order to give more detailed about the present work and reader will come to know that why developed method is much better as compare to well-known method.

Keywords: Nonlinear equations, Newton Raphson method, order of convergence.

I. INTRODUCTION

In this paper, we have developed an iterative method in order to find a simple root x* of the nonlinear equations

$$f(x) = 0$$

where f: D⊂R→R is a scalar function on an open interval D. The design of iterative formulae for solving nonlinear equations f(x) is a very interesting and important work in numerical analysis. Many iterative methods have been developed using different techniques including Taylor series, decomposition method, homotopy techniques, quadrature formulae [1-7]. It is observed repeatedly that the classical Newton Raphson method is one of the best iterative methods for solving the nonlinear equations

$$x_{n+1} = x_n - \frac{f(x_n)}{f`(x_n)}$$

This method is fast converging and second quadratically converge technique but is not reliable as it has pitfall [8]. However, it is most useful and vigorous numerical techniques. Recently, it is seen in literature that a few modifications in Newton Raphson method has been observed using Taylor series and difference operator for finding a single root of a nonlinear equation [9-10]. Consequently, we have suggested a new iterated method using references [11-13] and Newton Raphson Method to find the real root of nonlinear equations. The purpose of new iterated method is to introduce a mathematical tool for solving all possible roots of polynomials of higher degree functions and transcendental functions. It is shown that the proposed method has second order of convergence. C++ and MATLAB have been used to explain the results of second order iterated method.

II. ITERATIVE METHOD

By using the Newton Raphson Method, such as

$$x = x_0 - \frac{f(x_0)}{f`(x_0)}$$

For better approximation, we are taking average of $\frac{1}{f`(x)}$ at point (x_0, x_1), we get

$$x = x_0 - \frac{f(x_0)}{2}\left[\frac{1}{f`(x_0)} + \frac{1}{f`(x_1)}\right]$$

In general,

$$x = x_n - \frac{f(x_n)}{2}\left[\frac{1}{f`(x_n)} + \frac{1}{f`(x_{n+1})}\right]$$

This is homerier method, where x_0 is an initial guess sufficiently close to root. The homerier method can also be written as

$$x = x_{n+1} - \frac{f(x_{n+1})}{2}\left[\frac{1}{f`(x_{n+1})} + \frac{1}{f`(x_n)}\right]$$

Finally, we get

$$x_{n+1} = y_n - \frac{f(y_n)}{2}\left[\frac{1}{f`(y_n)} + \frac{1}{f`(x_n)}\right]$$

Where $y_n = x_n - \frac{f(x_n)}{f`(x_n)}$

Hence this is new iterated method.

III. CONVERGENCE ANALYSIS

The following statement will be used to show that the New Developed Method has second order of Convergence.

Proof:

Using the relation $e_n = x_n - \mathrm{a}$ in Taylor series, therefore from Taylor series we estimate $f(x_n), f`(x_n)$ *and* $f\left(x_n - \frac{f(x_n)}{f`(x_n)}\right)$ with using this condition $\mathrm{c} = \frac{f``(a)}{2f`(a)}$ and neglecting higher order terms in order to obtain required condition. Such as

$$f(x_n) = f`(a)(e_n + ce_n{}^2) \quad --- (i)$$

$$f`(x_n) = f`(a)(1 + 2ce_n) \quad --- (ii)$$

By using (i) *and* (ii) *in* (2), we get

$$y_n = e_n - \frac{f`(a)(e_n + ce_n{}^2)}{f`(a)(1 + 2ce_n)}$$

$$y_n = e_{n-}e_n(1 + ce_n)(1 + 2ce_n)^{-1}$$

$$y_n = e_{n-}e_n(1 + ce_n)(1 - 2ce_n)$$

$$y_n = e_{n-}e_n(1 - ce_n)$$

$$y_n = \mathrm{c}e^2{}_n$$

Thus,

$$f(y_n) = f`(a)(\mathrm{c}e^2{}_n + c^3 e_n{}^4)$$

$$f(y_n) = \mathrm{c}e^2{}_n f`(a)(1 + c^2 e_n{}^2) \quad --- (iii)$$

And

$$f'(y_n) = f`(a)(2\mathrm{c}e_n + 3c^3 e_n{}^3)$$

$$f'(y_n) = \mathrm{c}e_n f`(a)(2 + 3c^2 e_n{}^2) \quad -- (iv)$$

By using (i), (ii), (iii) *and* (iv) *in* (3), we get

$$e_{n+1} = \mathrm{c}e^2{}_n - \frac{ce^2{}_n f`(a)(1 + c^2 e_n{}^2)}{2}\left[\frac{1}{ce_n f`(a)(2 + 3c^2 e_n{}^2)} + \frac{1}{f`(a)(1 + 2ce_n)}\right]$$

$$e_{n+1} = \mathrm{c}e^2{}_n - \frac{\mathrm{c}e^2{}_n(1 + c^2 e_n{}^2)}{2}\left[\frac{1}{\mathrm{c}e_n(2 + 3c^2 e_n{}^2)} + \frac{1}{(1 + 2ce_n)}\right]$$

$$e_{n+1} = \mathrm{c}e^2{}_n - \frac{\mathrm{c}e^2{}_n(1 + c^2 e_n{}^2)}{2}\left[\frac{1 + 2ce_n + \mathrm{c}e_n(2 + 3c^2 e_n{}^2)}{\mathrm{c}e_n(2 + 3c^2 e_n{}^2)(1 + 2ce_n)}\right]$$

$$e_{n+1} = \mathrm{c}e^2{}_n - \frac{e_n(1 + c^2 e_n{}^2)}{2}\left[\frac{1 + 4ce_n}{(2 + 4ce_n + 3c^2 e_n{}^2)}\right]$$

$$e_{n+1} = \mathrm{c}e^2{}_n - \frac{e_n(1 + c^2 e_n{}^2)}{4}[(1 + 4ce_n)(1 + 4ce_n + 3c^2 e_n{}^2)^{-1}]$$

$$e_{n+1} = \mathrm{c}e^2{}_n - \frac{e_n(1 + c^2 e_n{}^2)}{4}[(1 + 4ce_n)(1 - 4ce_n - 3c^2 e_n{}^2)]$$

$$e_{n+1} = \frac{4\mathrm{c}e^2{}_n - e_n(1 + c^2 e_n{}^2)(1 - 16c^2 e_n{}^2)}{4}$$

$$e_{n+1} = \frac{4\mathrm{c}e^2{}_n - e_n(1 - 15c^2 e_n{}^2)}{4}$$

$$e_{n+1} = \frac{-e_n + 4\mathrm{c}e^2{}_n + 15c^2 e_n{}^3}{4} \quad --- (v)$$

for nonlinear equation such as $f(x_n) = 0$, *then* (i) can also be written as

$$f`(a)e_n + f``(a)e_n{}^2 = 0$$

$$e_n = \frac{-f``(a)e_n{}^2}{f`(a)}$$

$$e_n = ce_n{}^2 \quad --- (vi)$$

Note $c = \frac{-f``(a)}{f`(a)}$

By using (vi) *in* (v), we get

$$e_{n+1} = \frac{-ce_n{}^2 + 4ce^2{}_n + 15c^2e_n{}^3}{4}$$

$$e_{n+1} = \frac{3}{4}ce^2{}_n + \frac{15}{4}c^2e_n{}^3 \qquad --- \; (vii)$$

Hence this proves that the proposed iterative method has second order of convergence.

IV. RESULTS AND DISCUSSION

In this section, we have given a few examples in order to show that why our developed method is much better as compare to well-know method. These results are test by Newton Raphson Method. Numerical results are solved by using C++ programing and all calculations have been completed. We are using the accuracy criteria such as $|x_{n+1} - x_n| < 10^{10}$. In Table-1, we have listed the results obtained by new method and compared with Newton Raphson Method. As we can observe from the Table-1 that the results obtained by our method is much better as it converges to the root much faster than the Newton Raphson, and in Table-1, it is also shown that the number of iterations of each method using different initial guess. Henceforth, from results it has been observed that the table-1 clearly shows that proposed method takes less number of iterations and gives better accuracy than Newton Raphson Method for solving nonlinear equations.

TABLE-1

FUNCTIONS	METHODS	ITERATIONS	X	A E %
$\sin^2x-x^2+1$	Newton Raphson Method	5	1.40449	5.96046e^{-7}
x_0=1	Newton-type Method	4		1.44243e^{-5}
2x-lnx-7	Newton Raphson Method	4	4.21991	4.76837e^{-5}
x_0=6	Newton-type Method	3		1.37329e^{-4}
e^{-x}-cosx	Newton Raphson Method	3	4.72129	1.21212e^{-3}
x_0=4	Newton-type Method	3		1.57356e^{-5}
$2x^2$ -5x-2	Newton Raphson Method	5	0.350781	2.98023e^{-8}
x_0=0	Newton-type Method	3		1.15931e^{-5}
Sinx-x+1	Newton Raphson Method	5	1.93456	5.84126e^{-6}
x_0=1	Newton-type Method	4		6.85453e^{-5}

V. CONCLUSION

We have composed a Newton-type iterative method with second order convergence obtained with Newton raphson method and numerical technique for estimating nonlinear equations. From Table-1, it has been shown that the efficiency of proposed method performs better than classical second order newton raphson method as iteration point of view and as well as accuracy perceived. Henceforth, from numerical results demonstrate that the proposed method is performing-well, more efficient and easy to employ with reliable results for solving non-linear equations.

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